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Some Issues Concerning a Version of the Theory of Thin Solids Based on Expansions in a System of Chebyshev Polynomials of the Second Kind

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Abstract—We consider various forms of equations of motion and heat influx for deformable solids as well as various forms of Hooke's law and Fourier's heat conduction law under the nonclassical parametrization [1–5] of the domain occupied by a thin solid, where the transverse coordinate ranges in the interval $[0, 1]$. We write out several characteristics inherent in this parametrization. We use the above-mentioned equations and laws to derive the corresponding equations and laws, as well as statements of problems, for thin bodies in moments with respect to Chebyshev polynomials of the second kind. Here the interval $[0, 1]$ is used as the orthogonality interval for the systems of Chebyshev polynomials. For this interval, we write out the basic recursion relations and, in turn, use them to obtain several additional recursion relations, which play an important role in constructing other versions of the theory of thin solids. In particular, we use the recursion relations to obtain the moments of the first and second derivatives of a scalar function, of rank one and two tensors and their components, and of some differential operators of these variables. Moreover, we give the statements of coupled and uncoupled dynamic problems in moments of the (r, N)th approximation in moment thermomechanics of thin deformable solids. We also state the nonstable temperature problem in moments of the (r, N)th approximation.

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1. GENERATING FUNCTION. BASIC RECURSION RELATIONS FOR CHEBYSHEV POLYNOMIALS OF THE SECOND KIND

The polynomials

$$U_n(x) = \frac{1}{n+1} T'_{n+1}(x), \quad -1 \le x \le 1, \quad n \in \mathbb{N}_0, \tag{1.1}$$

where $\mathbb{N}_0$ is the set of nonnegative integers and the $T_n(x) = \cos(n \arccos x)$, $x \in [-1, 1]$, $n \in \mathbb{N}_0$, are Chebyshev polynomials of the first kind, are called Chebyshev polynomials of the second kind [6–10] on the interval $[-1, 1]$. They are orthogonal polynomials on this interval with weight $h(x) = \sqrt{1 - x^2}$.

By definition, these polynomials and the orthonormal polynomials $\hat{U}_n(x)$ can be represented as

$$\begin{aligned} &U_n(x) = \frac{\sin[(n+1)\arccos x]}{\sqrt{1-x^2}}, \quad \hat{U}_n(x) = \frac{1}{\|U_n\|} U_n(x), \\ &\|U_n\| = \sqrt{\frac{\pi}{2}}, \quad -1 \le x \le 1, \quad n \in \mathbb{N}_0. \end{aligned} \tag{1.2}$$

Using the generating function [6] $F(r, x) = 1/(1 - 2rx + r^2)$, $|r| < 1$, $|x| \le 1$, of Chebyshev polynomials of the second kind and proceeding, for example, as was done in [10] for Legendre polynomials, one can readily prove the formulas

$$\begin{aligned} &2xU_n(x) = U_{n-1}(x) + U_{n+1}(x), \quad xU'_n(x) = nU_n(x) + U'_{n-1}(x), \quad n \ge 1, \\ &U'_n(x) = 2nU_{n-1}(x) + U'_{n-2}(x), \quad n \ge 2. \end{aligned} \tag{1.3}$$

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Note that the first relation in (1.3) can be derived directly [10] from the first formula in (1.2).

Now consider the linear transformation $x = 2t - 1$, $0 \le t \le 1$, which takes the interval $[0, 1]$ to the interval $[-1, 1]$. Then, by the theorem on linear transformations of the orthogonality interval [10, 11], the system of polynomials

$$U_n^*(t) \equiv U_n(2t-1) = \frac{\sin[(n+1)\arccos(2t-1)]}{2\sqrt{t(1-t)}}, \quad 0 \le t \le 1, \quad n \in \mathbb{N}_0, \tag{1.4}$$

which are called shifted Chebyshev polynomials of the second kind, is orthogonal.

Hence, by analogy with the second formula in (1.2), the orthonormal shifted Chebyshev polynomials of the second kind can be represented as

$$\hat{U}_n^*(t) = \frac{1}{\|U_n^*\|}U_n^*(t) = \frac{1}{\|U_n^*\|}\frac{\sin[(m+1)\arccos(2t-1)]}{2\sqrt{t(1-t)}}, \quad \|U_n^*\| = \frac{\sqrt{\pi}}{2}, \quad n \in \mathbb{N}_0. \tag{1.5}$$

Further, proceeding as above and using the generating function $F^*(r,t) = 1/[(1+r)^2 - 4rt]$, $|r| < 1$, $t \in [0, 1]$, we can obtain the basic recursion relations for shifted Chebyshev polynomials of the second kind (1.4) on the interval $[0, 1]$. We have

$$\begin{aligned} &4tU_n^*(t) = U_{n-1}^*(t) + 2U_n^*(t) + U_{n+1}^*(t), \quad n \ge 1,\\ &2tU_n^{*\prime}(t) = 2nU_n^*(t) + U_{n-1}^{*\prime}(t) + U_n^{*\prime}(t), \quad n \ge 1,\\ &U_n^{*\prime}(t) = 4nU_{n-1}^*(t) + U_{n-2}^{*\prime}(t), \quad n \ge 2. \end{aligned} \tag{1.6}$$

2. ADDITIONAL RECURSION RELATIONS FOR CHEBYSHEV POLYNOMIALS OF THE SECOND KIND ON THE INTERVAL $[0, 1]$

These recursion relations can be obtained from (1.6) in the same way as some of their analogs for Legendre polynomials in [11]. Therefore, we do not derive them here but only write out the relations used most frequently:

$$2^{2s}t^sU_k^*(t) = \sum_{p=0}^{2s}\binom{2s}{p}U_{k-s+p}^*(t), \quad k - s \ge 0, \quad k \in \mathbb{N}_0, \tag{2.1}$$

$$2^{2(k+1)}t^{k+1}U_k^*(t) = \sum_{p=1}^{2k+2}\binom{2k+2}{p}U_{p-1}^*(t), \quad k \in \mathbb{N}_0, \tag{2.2}$$

$$2^{2(k+s)}t^{k+s}U_k^*(t) = -\sum_{q=2}^{s}\binom{2k+2s}{q-2}U_{s-q}^*(t) + \sum_{p=s}^{2k+2s}\binom{2k+2s}{p}U_{p-s}^*(t), \quad k \in \mathbb{N}_0, \quad s \ge 2, \tag{2.3}$$

$$2^{2s}t^sU_m^*(t)U_n^*(t) = \sum_{p=0}^{m}\sum_{q=0}^{2s}\binom{2s}{q}U_{n-m-s+2p+q}^*(t), \quad n - m - s \ge 0, \tag{2.4}$$

$$U_n^{*\prime}(t) = 4\sum_{k=0}^{[(n-1)/2]}(n-2k)U_{n-(2k+1)}^*(t) = 4\sum_{k=0}^{[(n-1)/2]}(2k+1+a)U_{2k+a}^*(t), \quad n \ge 1, \tag{2.5}$$

$$\begin{aligned} U_n^{*\prime\prime}(t) &= 2^4\sum_{k=0}^{[(n-2)/2]}(k+1)(n-k)[n-(2k+1)]U_{n-(2k+2)}^*(t)\\ &= 2^2\sum_{k=0}^{[(n-2)/2]}(2k+2-a)[(n+1)^2-(2k+2-a)^2]U_{2k+1-a}^*(t), \quad n \ge 2, \end{aligned} \tag{2.6}$$

$$\begin{aligned} 2^{2s}t^s U_n^{*\prime}(t) &= 4\sum_{k=0}^{(n-s-2)/2}\sum_{p=0}^{2s}(n-2k)\binom{2s}{p}U_{p-u}^*(t) + 4s\sum_{p=1}^{2s}\binom{2s}{p}U_{p-1}^*(t) \\ &\quad + 4\sum_{k=(n-s+2)/2}^{[(n-1)/2]}(n-2k)\Big[-\sum_{q=2}^{u}\binom{2s}{q-2}U_{u-q}^*(t) + \sum_{p=u}^{2s}\binom{2s}{p}U_{p-u}^*(t)\Big], \end{aligned} \tag{2.7}$$

$u = 2k+1+s-n, \quad n-s = 2l, \quad l \geq 0, \quad n \geq 1, \quad s \geq 0,$

$$\begin{aligned} 2^{2s}t^s U_n^{*\prime}(t) &= 4\sum_{k=0}^{(s-a-2)/2}(2k+1+a)\Big[-\sum_{q=2}^{v}\binom{2s}{q-2}U_{v-q}^*(t) + \sum_{p=v}^{2s}\binom{2s}{p}U_{p-v}^*(t)\Big] \\ &\quad + 4\sum_{k=(s-a)/2}^{[(n-1)/2]}\sum_{p=0}^{2s}(2k+1+a)\binom{2s}{p}U_{p-v}^*(t), \end{aligned} \tag{2.8}$$

$v = s-2k-a, \quad n-s = 2l+1, \quad l \geq 0, \quad n \geq 1, \quad s \geq 0,$

$$2^{2s}t^s U_n^{*\prime}(t) = 4\sum_{k=0}^{[(n-1)/2]}(2k+1+a)\Big[-\sum_{q=2}^{v}\binom{2s}{q-2}U_{v-q}^*(t) + \sum_{p=v}^{2s}\binom{2s}{p}U_{p-v}^*(t)\Big], \tag{2.9}$$

$v = s-2k-a, \quad s-n \geq 1, \quad n \geq 1,$

$$\begin{aligned} 2^{2s}t^s U_n^{*\prime\prime}(t) &= 2^4\sum_{k=0}^{(n-s-2)/2}\sum_{p=0}^{2s}(k+1)(n-k)[n-(2k+1)]\binom{2s}{p}U_{p-r}^*(t), \\ &\quad + 2^4\sum_{k=(n-s)/2}^{[(n-2)/2]}(k+1)(n-k)[n-(2k+1)]\Big[-\sum_{q=2}^{r}\binom{2s}{q-2}U_{r-q}^*(t) + \sum_{p=r}^{2s}\binom{2s}{p}U_{p-r}^*(t)\Big], \end{aligned} \tag{2.10}$$

$r = 2k+2+s-n, \quad n \geq 2, \quad n-s = 2l, \quad l \geq 0, \quad s \geq 0,$

$$\begin{aligned} 2^{2s}t^s U_n^{*\prime\prime}(t) &= 2^4\sum_{k=0}^{(n-s-3)/2}\sum_{p=0}^{2s}(k+1)(n-k)[n-(2k+1)]\binom{2s}{p}U_{p-r}^*(t) \\ &\quad + 2^2 s[(n+1)^2 - s^2]\sum_{p=1}^{s}\binom{2s}{p}U_{p-1}^*(t) \\ &\quad + 2^4\sum_{k=(n-s+1)/2}^{[(n-2)/2]}(k+1)(n-k)[n-(2k+1)]\Big[-\sum_{q=2}^{r}\binom{2s}{q-2}U_{r-q}^*(t) + \sum_{p=r}^{2s}\binom{2s}{p}U_{p-r}^*(t)\Big], \end{aligned} \tag{2.11}$$

$r = 2k+2+s-n, \quad n \geq 2, \quad n-s = 2l+1, \quad l \geq 0, \quad s \geq 0,$

$$\begin{aligned} 2^{2s}t^s U_n^{*\prime\prime}(t) &= 2^2\sum_{k=0}^{[(n-2)/2]}(2k+2-a)[(n+1)^2-(2k+2-a)^2]\Big[-\sum_{q=2}^{m}\binom{2s}{q-2}U_{m-q}^*(t) \\ &\quad + \sum_{p=m}^{2s}\binom{2s}{p}U_{p-m}^*(t)\Big], \quad m = s-2k-1+a, \quad n \geq 2, \quad s-n \geq 1. \end{aligned} \tag{2.12}$$

Here $a = n-1-2[(n-1)/2]$, $[x]$ is the integer part of a number x, and $\binom{m}{n}$ are the binomial coefficients. It should be noted that all relations (2.2)–(2.12), which are also given in [12] and, except for (2.4), remain valid for the system $\{\hat{U}_k^*\}_{k=0}^{\infty}$ of orthonormal Chebyshev polynomials of the second kind, can be proved by induction.

For the system of orthonormal polynomials, (2.4) can be represented as

$$2^{2s}t^s\hat{U}^*_m(t)\hat{U}^*_n(t) = \hat{U}^*_0\sum_{p=0}^{m}\sum_{q=0}^{2s}\binom{2s}{q}\hat{U}^*_{n-m-s+2p+q}(t), \quad n-m-s\ge 0. \tag{2.13}$$

A relation similar to (2.2) for Chebyshev polynomials of first kind is given in [13] as an exercise. The relations obtained from (2.4) for $s=0$ and $m=n-1$, as well as for $s=0$ and $m=n=k-1$, can also be found there.

3. REPRESENTATION OF SOME DIFFERENTIAL OPERATORS APPLIED TO TENSORS AND OF THE EQUATIONS OF MOTION IN MECHANICS OF DEFORMABLE SOLIDS

We consider the representations of the gradient, divergence, rotor, repeated gradient, Laplacian, and repeated divergence of a tensor, as well as the equations of motion in stresses and couple stresses for the new parametrization of the domain occupied by a thin solid. Prior to finding these representations, we recall basic facts concerning the parametrization of the domain occupied by a thin solid [1–5]. The new parametrization consists in determining the position vector of an arbitrary point of the domain occupied by the thin solid in the form

$$\mathbf{r}(x',x^3) = (1-x^3)\overset{(-)}{\mathbf{r}}(x') + x^3\overset{(+)}{\mathbf{r}}(x') = \overset{(-)}{\mathbf{r}}(x') + x^3\mathbf{h}(x'), \quad x'=(x^1,x^2), \quad x^3\in[0,1],$$

where $\overset{(-)}{\mathbf{r}}(x')$ and $\overset{(+)}{\mathbf{r}}(x')$, respectively, determine the interior $\overset{(-)}{S}$ and exterior $\overset{(+)}{S}$ base surfaces of the thin solid. In our case, the vector $\mathbf{h}(x') = \overset{(+)}{\mathbf{r}}(x') = \overset{(-)}{\mathbf{r}}(x')$, taking the interior base surface $\overset{(-)}{S}$ onto the exterior base surface $\overset{(+)}{S}$ is assumed to be perpendicular to the interior base surface. The basis vectors $\mathbf{r}_p$ and $\mathbf{r}^p$ at an arbitrary point of the domain occupied by the thin solid and the basis vectors $\mathbf{r}_{m^-}$, $\mathbf{r}^{m^-}$ and $\mathbf{r}_{m^+}$, $\mathbf{r}^{m^+}$ at the corresponding points on the interior and exterior base surfaces are related to each other by the transition components[1)] of the unit tensor of the second rank [1–5]:

$$\mathbf{r}_p = g_p^{m^-}\mathbf{r}_{m^-} = g_{pm^+}\mathbf{r}^{m^+}, \quad \mathbf{r}^p = g^p_{m^-}\mathbf{r}^{m^-} = g^p_{m^+}\mathbf{r}^{m^+}. \tag{3.1}$$

Relations (3.1) remain valid when the indices are raised or lowered. Next, note that

$$\begin{aligned} &g^P_{M^-} = \overset{(-)}{\vartheta}{}^{-1}A^P_{M^-}, \quad \overset{(-)}{\vartheta} = \det(g_I^{J^-}), \quad g^3_{M^-} = -g_P^{3^-}g^P_{M^-}, \quad g_P^{3^-} = x^3g_{P^+}^{3^-},\\ &g_{P^+}^{3^-} = \partial_P\ln h, \quad h=|\mathbf{h}|, \quad A^P_{M^-}\equiv g^{P^-}_{M^-} + x^3a^P_{M^+}, \quad a^P_{M^+}\equiv (g^{I^-}_{I^+}-1)g^{P^-}_{M^-} - g^{P^-}_{M^+}. \end{aligned} \tag{3.2}$$

Owing to the first and second relations in (3.2), from the second relation in (3.1) we obtain

$$\mathbf{r}^P = g^P_{M^-}\mathbf{r}^{M^-}, \quad \mathbf{r}^3 = g^3_{M^-}\mathbf{r}^{M^-} + \mathbf{r}^{3^-} = \mathbf{r}^{3^-} - g_P^{3^-}\mathbf{r}^P = \mathbf{r}^{3^-} - g_P^{3^-}g^P_{M^-}\mathbf{r}^{M^-}. \tag{3.3}$$

3.1. Representations of gradient, divergence, and rotor

The repeated gradient operator can be applied to any tensor. Therefore, denoting some tensor quantity by $\mathbb{F}(x',x^3)$, by the definition of gradient [14–17] and from (3.3) we have

$$\operatorname{grad}\mathbb{F} = \nabla\mathbb{F} = \mathbf{r}^p\partial_p\mathbb{F} = \mathbf{r}^P\partial_P\mathbb{F} + \mathbf{r}^3\partial_3\mathbb{F} = \mathbf{r}^P(\partial_P - g_P^{3^-}\partial_3)\mathbb{F} + \mathbf{r}^{3^-}\partial_3\mathbb{F} = \mathbf{r}^{M^-}g^P_{M^-}(\partial_P - g_P^{3^-}\partial_3)\mathbb{F} + \mathbf{r}^{3^-}\partial_3\mathbb{F}.$$

Thus, introducing the differential operator

$$N_p = \partial_p - g_p^{3^-}\partial_3, \quad N = \mathbf{r}^pN_p = \mathbf{r}^PN_P = \mathbf{r}^{M^-}g^P_{M^-}N_P, \quad N_3 = 0, \tag{3.4}$$

we obtain the desired representation of the gradient in the form

$$\operatorname{grad}\mathbb{F} = \nabla\mathbb{F} = \mathbf{r}^PN_P\mathbb{F} + \mathbf{r}^{3^-}\partial_3\mathbb{F} = \mathbf{r}^{M^-}g^P_{M^-}N_P\mathbb{F} + \mathbf{r}^{3^-}\partial_3\mathbb{F}. \tag{3.5}$$

[1)]The conventional rules of tensor calculus [14–17] are used. We preserve the main notation and relations used in earlier papers.

The divergence operator can be applied to a tensor whose rank is greater than or equal to 1. Applying this operator, for example, to a rank two tensor $\underset{\sim}{\mathbf{P}}$ and using its definition, the third relation in (3.2), and (3.4), we obtain

$$\begin{aligned}\operatorname{div}\underset{\sim}{\mathbf{P}} &= \nabla\cdot\underset{\sim}{\mathbf{P}} = \mathbf{r}^p\cdot\partial_p\underset{\sim}{\mathbf{P}} = \nabla_p\mathbf{P}^p = \nabla_p(g^p_{m^-}\mathbf{P}^{m^-}) = g^p_{m^-}\nabla_p\mathbf{P}^{m^-}\\ &= g^P_{M^-}\nabla_P\mathbf{P}^{M^-} + g^3_{M^-}\nabla_3\mathbf{P}^{M^-} + \partial_3\mathbf{P}^{3^-} = g^P_{M^-}(\nabla_P - g^{3^-}_P\nabla_3)\mathbf{P}^{M^-} + \partial_3\mathbf{P}^{3^-} = g^P_{M^-}N_P\mathbf{P}^{M^-} + \partial_3\mathbf{P}^{3^-};\end{aligned}$$

i.e., the desired representation of the divergence operator of the tensor of the second kind has the form

$$\operatorname{div}\underset{\sim}{\mathbf{P}} = g^P_{M^-}N_P\mathbf{P}^{M^-} + \partial_3\mathbf{P}^{3^-} \tag{3.6}$$

It is easy to see that the expression for the divergence of the vector $\mathbf{u}$ in (3.6) can be obtained if $\underset{\sim}{\mathbf{P}}$ and $\mathbf{P}^{M^-}$ in this relation are replaced by $\mathbf{u}$ and $\mathbf{u}^{M^-}$, respectively. We also note that (3.6) readily follows from (3.5) if $\mathbb{F}$ in it is replaced by $\underset{\sim}{\mathbf{P}} = \mathbf{r}_{n^-}\mathbf{P}^{n^-}$ and the tensor multiplication symbol, which is omitted by agreement, is replaced by the scalar multiplication symbol.

The rotor operator can also be applied to a tensor of rank greater than or equal to 1. Its representation can be obtained by analogy with (3.6) or from (3.5). We derive the representation of $\operatorname{rot}\mathbf{u}$ from (3.5). Using the definition of rotor, it suffices to replace $\mathbb{F}$ in (3.5) by $\mathbf{u}$ and replace the tensor multiplication symbol by the vector multiplication symbol. After transformations, we obtain the desired representation in the form

$$\operatorname{rot}\mathbf{u} = \nabla\times\mathbf{u} = C^{L^-M^-}(g^P_{M^-}N_Pu_{3^-} - \nabla_3u_{M^-})\mathbf{r}_{L^-} + C^{M^-N^-}g^P_{M^-}N_Pu_{N^-}\mathbf{r}_{3^-} \tag{3.7}$$

Here $C^{M^-N^-} = (\mathbf{r}^{M^-}\times\mathbf{r}^{N^-})\cdot\mathbf{r}^{3^-}$ are the components of the discriminant tensor.

3.2. Representations of repeated gradient, Laplacian, and repeated divergence

The gradient operator can be applied to a tensor of any rank. Applying it to some tensor symbol $\mathbb{F}$, by (3.5), we obtain

$$\operatorname{grad}\operatorname{grad}\mathbb{F} = \nabla\nabla\mathbb{F} = \mathbf{r}^{M^-}g^P_{M^-}N_P(\nabla\mathbb{F}) + \mathbf{r}^{3^-}\partial_3(\nabla\mathbb{F}). \tag{3.8}$$

Using the third relation in (3.4), we obtain

$$\begin{aligned}N_P(\nabla\mathbb{F}) &= N_P(\mathbf{r}^q\partial_q\mathbb{F}) = N_P(\mathbf{r}^{n^-}g^q_{n^-}\partial_q\mathbb{F}) = \mathbf{r}^{n^-}N_P(g^q_{n^-}\partial_q\mathbb{F})\\ &= \mathbf{r}^{N^-}N_P(g^q_{N^-}\partial_q\mathbb{F}) + \mathbf{r}^{3^-}N_P(\partial_3\mathbb{F}) = \mathbf{r}^{N^-}N_P(g^Q_{N^-}\partial_Q\mathbb{F}) + \mathbf{r}^{3^-}N_P(\partial_3\mathbb{F}).\end{aligned}$$

We represent the first term on the right-hand side in the last relation in a different form. According to (3.4), we obtain

$$N_P(g^Q_{N^-}N_Q\mathbb{F}) = N_P(g^q_{N^-}N_q\mathbb{F}) = g^q_{N^-}N_P(N_q\mathbb{F}) = g^Q_{N^-}N_P(N_Q\mathbb{F}) + g^3_{N^-}N_P(N_3\mathbb{F}) = g^Q_{N^-}N_P(N_Q\mathbb{F}).$$

Thus,

$$N_P(\nabla\mathbb{F}) = \mathbf{r}^{N^-}N_P(g^Q_{N^-}N_Q\mathbb{F}) + \mathbf{r}^{3^-}N_P(\partial_3\mathbb{F}) = \mathbf{r}^{N^-}g^Q_{N^-}N_PN_Q\mathbb{F} + \mathbf{r}^{3^-}N_P\partial_3\mathbb{F}. \tag{3.9}$$

In a similar way, according to the third relation in (3.4), we obtain

$$\partial_3(\nabla\mathbb{F}) = \mathbf{r}^{N^-}\partial_3(g^Q_{N^-}N_Q\mathbb{F}) + \mathbf{r}^{3^-}\partial_3^2\mathbb{F} = \mathbf{r}^{N^-}g^Q_{N^-}\nabla_3N_Q\mathbb{F} + \mathbf{r}^{3^-}\partial_3^2\mathbb{F}. \tag{3.10}$$

Taking into account (3.9) and (3.10), from (3.8) we obtain two desired representations of the repeated gradient operator, one of which has the form

$$\nabla\nabla\mathbb{F} = \mathbf{r}^{M^-}\mathbf{r}^{N^-}g^P_{M^-}g^Q_{N^-}N_PN_Q\mathbb{F} + \mathbf{r}^{M^-}\mathbf{r}^{3^-}g^P_{M^-}N_P\partial_3\mathbb{F} + \mathbf{r}^{3^-}\mathbf{r}^{N^-}g^Q_{N^-}\nabla_3N_Q\mathbb{F} + \mathbf{r}^{3^-}\mathbf{r}^{3^-}\partial_3^2\mathbb{F}. \tag{3.11}$$

Let us give another, more expanded form of the representation of this operator. It follows from (3.3) that, after transformations, we obtain

$$\nabla\nabla\mathbb{F} = \mathbf{r}^{M^-}\mathbf{r}^{N^-}g^P_{M^-}g^Q_{N^-}[\nabla_P\nabla_Q - (\partial^{3^-}_P\nabla_3\nabla_Q + g^{3^-}_Q\nabla_P\nabla_3) + g^{3^-}_P g^{3^-}_Q\nabla_3\nabla_3]\mathbb{F}$$

$$+[\mathbf{r}^{M^-}\mathbf{r}^{3^-}g^P_{M^-}N_P\nabla_3+\mathbf{r}^3\mathbf{r}^{N^-}g^Q_{N^-}\nabla_3N_Q+\mathbf{r}^{3^-}\mathbf{r}^{3^-}\nabla_3\nabla_3]\mathbb{F}. \tag{3.12}$$

If the representations (3.11) and (3.12) of the repeated gradient operator are known, then this operator readily implies representations of the Laplacian and repeated divergence. Indeed, the Laplacian (the gradient divergence) can be applied to any quantity. We find its representation from (3.12) if, on the left-hand side between the Hamiltonian nabla-operators and on the right-hand side between the basis vectors, we replace the tensor multiplication symbol by the scalar multiplication symbol. As a result, we obtain

$$\Delta\mathbb{F}=\nabla\cdot\nabla\mathbb{F}=g^{M^-N^-}g^P_{M^-}g^Q_{N^-}[\nabla_P\nabla_Q-(g^{3^-}_P\nabla_3\nabla_Q+g^{3^-}_Q\nabla_P\nabla_3)+g^{3^-}_P g^{3^-}_Q\nabla_3^2]\mathbb{F}+g^{3^-3^-}\nabla_3^2\mathbb{F}. \tag{3.13}$$

When deriving (3.13), we have taken into account the fact that$g^{M^-3^-}=\mathbf{r}^{M^-}\cdot\mathbf{r}^{M^-}=0$.

The repeated divergence operator is applied to a tensor whose rank is greater than 1. Therefore, by taking, say, a tensor $\underset{\sim}{\mathbf{P}}$ of the second rank for $\mathbb{F}$ and by using the definition of this operator, from (3.12) we obtain

$$\begin{aligned}\operatorname{div}\operatorname{div}\underset{\sim}{\mathbf{P}}=\nabla\cdot(\nabla\cdot\underset{\sim}{\mathbf{P}})&=g^P_{M^-}g^Q_{N^-}[\nabla_P\nabla_Q-(g^{3^-}_P\nabla_3\nabla_Q+g^{3^-}_Q\nabla_P\nabla_3)+g^{3^-}_P g^{3^-}_Q\nabla_3^2]P^{N^-M^-}\\&+g^P_{M^-}N_P\nabla_3P^{3^-M^-}+g^Q_{N^-}\nabla_3N_QP^{N^-3^-}+\nabla_3^2P^{3^-3^-}.\end{aligned} \tag{3.14}$$

It should be noted that, by analogy with (3.13) and (3.14), from (3.12) one can obtain the representation of the repeated rotor operator and the inconsistency operator $\operatorname{Ink}\underset{\sim}{\mathbf{Q}}=\nabla\times(\nabla\times\underset{\sim}{\mathbf{P}})^T$, where $\underset{\sim}{\mathbf{Q}}$ is a tensor of the second rank. For brevity, we do not consider this in detail. We only note that $g^P_{M^-}$, $g^P_{M^-}g^Q_{N^-}$, and g^{PQ} can be represented [12] via x^3 in the form of series,

$$\begin{aligned}&g^P_{M^-}=\sum_{s=0}^{\infty}\underset{(s)}{A}{}^{P^-}_{M^+}(x^3)^s,\quad g^P_{M^-}g^Q_{N^-}=\sum_{s=0}^{\infty}\underset{(s)}{B}{}^{P^-Q^-}_{M^+N^+},\\&g^{PQ}=g^P_{M^-}g^Q_{N^-}g^{M^-N^-}=\sum_{s=0}^{\infty}(s+1)g^{Q^-M^-}\underset{(s)}{A}{}^{P^-}_{M^+}(x^3)^s.\end{aligned} \tag{3.15}$$

Here we have introduced the notation

$$\begin{aligned}&\underset{(s)}{A}{}^{P^-}_{M^+}=(g^{P^-}_{N_1^-}-g^{P^-}_{N_1^+})\cdot(g^{N_1^-}_{N_2^-}-g^{N_1^-}_{N_2^+})\cdot\ldots\cdot(g^{N^-_{s-2}}_{N^-_{s-1}}-g^{N^-_{s-2}}_{N^+_{s-1}})\cdot(g^{N^-_{s-1}}_{M^-}-g^{N^-_{s-1}}_{M^+}),\\&\underset{(0)}{A}{}^{P^-}_{M^+}=g^{P^-}_{M^-},\quad \underset{(s)}{B}{}^{P^-Q^-}_{M^+N^+}=\sum_{r=0}^{s}\underset{(s-r)}{A}{}^{P^-}_{M^+}\underset{(r)}{A}{}^{Q^-}_{N^+}.\end{aligned} \tag{3.16}$$

3.3. Representations of equations of motion in moment mechanics of deformable solids

As is known [18–22], the equations of motion in stress and couple stress tensors can be represented as

$$\nabla\cdot\underset{\sim}{\mathbf{P}}+\rho\mathbf{F}=\rho\partial_t^2\mathbf{u},\quad \nabla\cdot\underset{\sim}{\boldsymbol{\mu}}+\underset{\approx}{\mathbf{C}}\cdot\cdot\underset{\sim}{\mathbf{P}}^T+\rho\mathbf{m}=\underset{\sim}{\mathbf{J}}\cdot\partial_t^2\boldsymbol{\varphi}. \tag{3.17}$$

Here $\underset{\sim}{\mathbf{P}}$ and $\underset{\sim}{\boldsymbol{\mu}}$ are the of true and couple stress tensors, $\underset{\approx}{\mathbf{C}}$ is the discriminant tensor (a tensor of third rank) [14], $\mathbf{u}$ is the displacement vector, $\boldsymbol{\varphi}$ is the vector of (intrinsic) rotation, ρ is the material density in the actual configuration, $\mathbf{F}$ is the mass force density, $\mathbf{m}$ is the the mass moment density, and $g=\det(g_{ij})$ is the determinant of the fundamental matrix. The superscript "T" denotes the operation of transposition. Following the argument used in [23] for the equations of classical mechanics of deformable solids (MDS) in the classical parametrization of the domain occupied by a thin solid, in our case from (3.17) we obtain the following form of the representation of equations of the moment MDS:

$$
\begin{aligned}
&\frac{1}{\sqrt{\overset{(-)}{g}}}\partial_P\big(\sqrt{\overset{(-)}{g}}\,\overset{(-)}{\vartheta}\mathbf{P}^P\big)+\partial_3\big(\overset{(-)}{\vartheta}\mathbf{P}^3\big)+\rho\overset{(-)}{\vartheta}\mathbf{F}=\rho\overset{(-)}{\vartheta}\partial_t^2\mathbf{u},\\
&\frac{1}{\sqrt{\overset{(-)}{g}}}\partial_P\big(\sqrt{\overset{(-)}{g}}\,\overset{(-)}{\vartheta}\boldsymbol{\mu}^P\big)+\partial_3\big(\overset{(-)}{\vartheta}\boldsymbol{\mu}^3\big)+\mathbf{C}\cdot\cdot\big(\overset{(-)}{\vartheta}\underset{\sim}{\mathbf{P}}^T\big)+\rho\overset{(-)}{\vartheta}\mathbf{m}=\overset{(-)}{\vartheta}\underset{\sim}{\mathbf{J}}\cdot\partial_t^2\boldsymbol{\varphi},\\
&\overset{(-)}{g}=\det(g_{m^-n^-}),\quad g_{m^-n^-}=\mathbf{r}_{m^-}\cdot\mathbf{r}_{n^-}.
\end{aligned}
\tag{3.18}
$$

It is easy to see that, by (3.6), we can rewrite Eqs. (3.17) in the form

$$
g^P_{M^-}N_P\mathbf{P}^{M^-}+\partial_3\mathbf{P}^3+\rho\mathbf{F}=\rho\partial_t^2\mathbf{u},\quad g^P_{M^-}N_P\boldsymbol{\mu}^{M^-}+\partial_3\boldsymbol{\mu}^{3^-}+\underset{\approx}{\mathbf{C}}\cdot\cdot\underset{\sim}{\mathbf{P}}^T+\rho\mathbf{m}=\underset{\sim}{\mathbf{J}}\cdot\partial_t^2\boldsymbol{\varphi}.
\tag{3.19}
$$

Moreover, we have

$$
g^P_{M^-}N_P\mathbf{P}^{M^-}=g^P_{m^-}N_P\mathbf{P}^{m^-}=N_P(g^P_{m^-}\mathbf{P}^{m^-})=N_P(g^P_{M^-}\mathbf{P}^{M^-})=N_P\mathbf{P}^P.
$$

Therefore, Eqs. (3.19) can be represented in the following forms:

$$
\begin{aligned}
&N_P\mathbf{P}^P+\partial_3\mathbf{P}^{3^-}+\rho\mathbf{F}=\rho\partial_t^2\mathbf{u},\quad N_P\boldsymbol{\mu}^P+\partial_3\boldsymbol{\mu}^{3^-}+\underset{\approx}{\mathbf{C}}\cdot\cdot\underset{\sim}{\mathbf{P}}^T+\rho\mathbf{m}=\underset{\sim}{\mathbf{J}}\cdot\partial_t^2\boldsymbol{\varphi},\\
&N_P(g^P_{M^-}\mathbf{P}^{M^-})+\partial_3\mathbf{P}^{3^-}+\rho\mathbf{F}=\rho\partial_t^2\mathbf{u},\quad N_P(g^P_{M^-}\boldsymbol{\mu}^{M^-})+\partial_3\boldsymbol{\mu}^{3^-}+\underset{\approx}{\mathbf{C}}\cdot\cdot\underset{\sim}{\mathbf{P}}^T+\rho\mathbf{m}=\underset{\sim}{\mathbf{J}}\cdot\partial_t^2\boldsymbol{\varphi}.
\end{aligned}
\tag{3.20}
$$

Multiplying each relation in (3.19) by $\overset{(-)}{\vartheta}$ and taking into account the first relation in (3.2), we obtain

$$
\begin{aligned}
&A^P_{M^-}N_P\mathbf{P}^{M^-}+\overset{(-)}{\vartheta}\partial_3\mathbf{P}^{3^-}+\rho\overset{(-)}{\vartheta}\mathbf{F}=\rho\overset{(-)}{\vartheta}\partial_t^2\mathbf{u},\\
&A^P_{M^-}N_P\boldsymbol{\mu}^{M^-}+\overset{(-)}{\vartheta}\partial_3\boldsymbol{\mu}^{3^-}+\underset{\approx}{\mathbf{C}}\cdot\cdot\big(\overset{(-)}{\vartheta}\underset{\sim}{\mathbf{P}}^T\big)+\rho\overset{(-)}{\vartheta}\mathbf{m}=\overset{(-)}{\vartheta}\underset{\sim}{\mathbf{J}}\cdot\partial_t^2\boldsymbol{\varphi}.
\end{aligned}
\tag{3.21}
$$

We note that (3.18)–(3.21) are different forms of the representation of the equations of the moment MDS (3.17) for the considered parametrization of the domain occupied by the thin solid. We refer to them as various representations of the equations of moment mechanics of thin deformable solids (moment MTDS) for the new parametrization of the domain occupied by the thin solid.

Taking into account the first relation in (3.15), we can rewrite Eq. (3.19) in the form

$$
\begin{aligned}
&\sum_{s=0}^{\infty}A^{P^-}_{M^+}(x^3)^sN_P\mathbf{P}^{M^-}+\partial_3\mathbf{P}^{3^-}+\rho\mathbf{F}=\rho\partial_t^2\mathbf{u},\\
&\sum_{s=0}^{\infty}A^{P^-}_{M^+}(x^3)^sN_P\boldsymbol{\mu}^{M^-}+\partial_3\boldsymbol{\mu}^{3^-}+\underset{\approx}{\mathbf{C}}\cdot\cdot\underset{\sim}{\mathbf{P}}^T+\rho\mathbf{m}=\underset{\sim}{\mathbf{J}}\cdot\partial_t^2\boldsymbol{\varphi}.
\end{aligned}
\tag{3.22}
$$

One can see that Eqs. (3.22) contain infinitely many terms. Therefore, they cannot be used in practice. Naturally, the approximated equations with finitely many terms must be considered. In this connection, we introduce the following definition.

Definition 3.1. The equations obtained from (3.19) with the first $r+1$ terms preserved in the expansion of $g^P_{M^-}$ are called equations of the rth approximation.

The latter can be written in the form

$$
\underset{(r)}{g}{}^P_{M^-}N_P\mathbf{P}^{M^-}+\partial_3\mathbf{P}^{3^-}+\rho\mathbf{F}=\rho\partial_t^2\mathbf{u},\quad \underset{(r)}{g}{}^P_{M^-}N_P\boldsymbol{\mu}^{M^-}+\partial_3\boldsymbol{\mu}^{3^-}+\underset{\approx}{\mathbf{C}}\cdot\cdot\underset{\sim}{\mathbf{P}}^T+\rho\mathbf{m}=\underset{\sim}{\mathbf{J}}\cdot\partial_t^2\boldsymbol{\varphi},
\tag{3.23}
$$

$$
\underset{(r)}{g}{}^P_{M^-}=\sum_{s=m}^{r}A^{P^-}_{M^+}(x^3)^m.
\tag{3.24}
$$

From (3.23), for $r=0$, we obtain the equations of the zeroth approximation, for $r=1$, we obtain the equations of the first approximation, etc.

3.4. Representation of the heat influx equation in the moment MTDS

In the general case, the heat influx equation in the moment MTDS can be written in the form [24]

$$-\nabla\cdot\mathbf{q}+\rho q-T\frac{d}{dt}(\underset{\sim}{\mathbf{a}}\cdot\cdot\,\underset{\sim}{\mathbf{P}}^T+\underset{\sim}{\mathbf{d}}\cdot\cdot\,\underset{\sim}{\boldsymbol{\mu}}^T)+W^*=\rho c_p\partial_t T, \tag{3.25}$$

where $\mathbf{q}$ is the external heat flux vector, q is the mass heat influx, T is temperature, $\underset{\sim}{\mathbf{a}}$ and $\underset{\sim}{\mathbf{d}}$ are the thermal expansion tensors, $\underset{\sim}{\mathbf{P}}\neq\underset{\sim}{\mathbf{P}}^T$ is the stress tensor, $\underset{\sim}{\boldsymbol{\mu}}\neq\underset{\sim}{\boldsymbol{\mu}}^T$ is the couple stress tensor, W^* is the scattering function, ρ is the medium density, and c_p is the heat capacity under constant pressure. If we consider a physically linear medium, then the nonlinearity in (3.25) manifests itself in the third term on the left-hand side. A similar picture also occurs in the special version of this equation, which is obtained from (3.25) for $\underset{\sim}{\mathbf{d}}=0$ [25]. In the last case, since both of the heat capacities c_p and c_v (the heat capacity under constant volume) cannot simultaneously be constant (independent of temperature), it is frequently assumed that the temperature T in this term is replaced by the temperature $T_0=\text{const}$. With this assumption taken into account, the desired representation of the heat influx equation, by analogy with (3.19), has the form

$$-g^P_{M^-}N_P q^{M^-}-\partial_3 q^{3^-}+\rho q-T_0\frac{d}{dt}(\underset{\sim}{\mathbf{a}}\cdot\cdot\,\underset{\sim}{\mathbf{P}}^T+\underset{\sim}{\mathbf{d}}\cdot\cdot\,\underset{\sim}{\boldsymbol{\mu}}^T)+W^*=\rho c_p\partial_t T. \tag{3.26}$$

If necessary, one can easily write out other relations similar to (3.18) and (3.21). Therefore, for brevity, we do not dwell upon this. We only note that, on the basis of (3.26), by analogy with (3.23), the heat influx equation in the rth approximation can be written as

$$-\underset{(r)}{g}{}^P_{M^-}N_P q^{M^-}-\partial_3 q^{3^-}+\rho q-T_0\frac{d}{dt}(\underset{\sim}{\mathbf{a}}\cdot\cdot\,\underset{\sim}{\mathbf{P}}^T+\underset{\sim}{\mathbf{d}}\cdot\cdot\,\underset{\sim}{\boldsymbol{\mu}}^T)+W^*=\rho c_p\partial_t T. \tag{3.27}$$

3.5. Representations of Hooke's law and Fourier's thermal conduction law

In the linear moment theory of elasticity, owing to the generalized Duhamel–Neumann principle [24, 25], Hooke's law in nonisothermal processes can be represented in the form

$$\underset{\sim}{\mathbf{P}}=\underset{\approx}{\mathbf{C}}\cdot\cdot(\underset{\sim}{\boldsymbol{\gamma}}-\underset{\sim}{\mathbf{a}}\vartheta)+\underset{\approx}{\mathbf{A}}\cdot\cdot(\underset{\sim}{\boldsymbol{\kappa}}-\underset{\sim}{\mathbf{d}}\vartheta),\quad \underset{\sim}{\boldsymbol{\mu}}=\underset{\approx}{\mathbf{D}}\cdot\cdot(\underset{\sim}{\boldsymbol{\kappa}}-\underset{\sim}{\mathbf{d}}\vartheta)+\underset{\approx}{\mathbf{B}}\cdot\cdot(\underset{\sim}{\boldsymbol{\gamma}}-\underset{\sim}{\mathbf{a}}\vartheta), \tag{3.28}$$

where $\underset{\sim}{\boldsymbol{\gamma}}=\nabla\mathbf{u}-\underset{\approx}{\mathbf{C}}\cdot\boldsymbol{\varphi}$ is the strain tensor in moment theory [21], $\underset{\sim}{\boldsymbol{\kappa}}=\nabla\boldsymbol{\varphi}$ is the torsion–bending tensor, $\underset{\approx}{\mathbf{C}}$, $\underset{\approx}{\mathbf{A}}$, $\underset{\approx}{\mathbf{D}}$, $\underset{\approx}{\mathbf{B}}$ are the material tensors of rank four, and ϑ is the temperature difference.

Taking into account the expression for $\underset{\sim}{\boldsymbol{\gamma}}$, we can rewrite (3.28) in the form

$$\begin{aligned}&\underset{\sim}{\mathbf{P}}=\underset{\approx}{\mathbf{C}}\cdot\cdot\nabla\mathbf{u}+\underset{\approx}{\mathbf{A}}\cdot\cdot\nabla\boldsymbol{\varphi}-\underset{\approx}{\mathbf{C}}\cdot\cdot\underset{\approx}{\mathbf{C}}\cdot\boldsymbol{\varphi}-\underset{\sim}{\mathbf{b}}\vartheta,\quad \underset{\sim}{\boldsymbol{\mu}}=\underset{\approx}{\mathbf{D}}\cdot\cdot\nabla\boldsymbol{\varphi}+\underset{\approx}{\mathbf{B}}\cdot\cdot\nabla\mathbf{u}-\underset{\approx}{\mathbf{B}}\cdot\cdot\underset{\approx}{\mathbf{C}}\cdot\boldsymbol{\varphi}-\underset{\sim}{\boldsymbol{\beta}}\vartheta,\\&\underset{\sim}{\mathbf{b}}=\underset{\approx}{\mathbf{C}}\cdot\cdot\underset{\sim}{\mathbf{a}}+\underset{\approx}{\mathbf{A}}\cdot\cdot\underset{\sim}{\mathbf{d}},\quad \underset{\sim}{\boldsymbol{\beta}}=\underset{\approx}{\mathbf{D}}\cdot\cdot\underset{\sim}{\mathbf{d}}+\underset{\approx}{\mathbf{B}}\cdot\cdot\underset{\sim}{\mathbf{a}}.\end{aligned} \tag{3.29}$$

Here $\underset{\sim}{\mathbf{b}}$ and $\underset{\sim}{\boldsymbol{\beta}}$ are the tensors of thermomechanical properties.

We note that the special case of the law (3.29) was considered in [21, 22], and more general relations can be found in [24, 26].

Now one can easily find the desired representations of Hooke's law (3.29) for the new parametrization of the domain occupied by the thin solid. Indeed, using the operator (3.4), after simple transformations, we see from (3.29) that

$$\begin{aligned}&\underset{\sim}{\mathbf{P}}=\underset{\approx}{\mathbf{C}}{}^{M^-}\cdot g^P_{M^-}N_P\mathbf{u}+\underset{\approx}{\mathbf{C}}{}^{3^-}\cdot\partial_3\mathbf{u}+\underset{\approx}{\mathbf{A}}{}^{M^-}\cdot g^P_{M^-}N_P\boldsymbol{\varphi}+\underset{\approx}{\mathbf{A}}{}^{3^-}\cdot\partial_3\boldsymbol{\varphi}-\underset{\approx}{\mathbf{C}}\cdot\cdot\underset{\approx}{\mathbf{C}}\cdot\boldsymbol{\varphi}-\underset{\sim}{\mathbf{b}}\vartheta,\\&\underset{\sim}{\boldsymbol{\mu}}=\underset{\approx}{\mathbf{D}}{}^{M^-}\cdot g^P_{M^-}N_P\boldsymbol{\varphi}+\underset{\approx}{\mathbf{D}}{}^{3^-}\cdot\partial_3\boldsymbol{\varphi}+\underset{\approx}{\mathbf{B}}{}^{M^-}\cdot g^P_{M^-}N_P\mathbf{u}+\underset{\approx}{\mathbf{B}}{}^{3^-}\cdot\partial_3\mathbf{u}-\underset{\approx}{\mathbf{B}}\cdot\cdot\underset{\approx}{\mathbf{C}}\cdot\boldsymbol{\varphi}-\underset{\sim}{\boldsymbol{\beta}}\vartheta,\\&\underset{\approx}{\mathbf{C}}{}^{m^-}=\underset{\approx}{\mathbf{C}}\cdot\mathbf{r}^{m^-},\quad \underset{\approx}{\mathbf{A}}{}^{m^-}=\underset{\approx}{\mathbf{A}}\cdot\mathbf{r}^{m^-},\quad \underset{\approx}{\mathbf{D}}{}^{m^-}=\underset{\approx}{\mathbf{D}}\cdot\mathbf{r}^{m^-},\quad \underset{\approx}{\mathbf{B}}{}^{m^-}=\underset{\approx}{\mathbf{B}}\cdot\mathbf{r}^{m^-}.\end{aligned} \tag{3.30}$$

Taking into account the first relation in (3.15), we easily see that relations (3.30) contain infinitely many terms. Therefore, they cannot be used in this form. In applications, approximate constitutive equations (CE) are usually used, i.e., relations whose can be represented by using finitely many terms. In this connection, we introduce the following definition.

Definition 3.2. Relations obtained from (3.30) under the condition that the first $s+1$ terms are preserved in the expansion of $g^{P}_{M^-}$ are called the constitutive equations (CE) of the sth approximation.

It is easily seen that the CE of the sth approximation, by analogy with Eqs. (3.23) and (3.27), can be represented as

$$\begin{aligned}
\underset{\sim}{\mathbf{P}}_{(s)} &= \underset{\approx}{\mathbf{C}}^{M^-}\cdot \underset{(s)}{g}{}^{P}_{M^-} N_P\mathbf{u} + \underset{\approx}{\mathbf{C}}^{3^-}\cdot \partial_3\mathbf{u} + \underset{\approx}{\mathbf{A}}^{M^-}\cdot \underset{(s)}{g}{}^{P}_{M^-} N_P\boldsymbol{\varphi} + \underset{\approx}{\mathbf{A}}^{3^-}\cdot \partial_3\boldsymbol{\varphi} - \underset{\approx}{\mathbf{C}}\cdot\cdot\, \underset{\approx}{\mathbf{C}}\cdot\boldsymbol{\varphi} - \underset{\sim}{\mathbf{b}}\vartheta,\\
\underset{\sim}{\boldsymbol{\mu}}_{(s)} &= \underset{\approx}{\mathbf{D}}^{M^-}\cdot \underset{(s)}{g}{}^{P}_{M^-} N_P\boldsymbol{\varphi} + \underset{\approx}{\mathbf{D}}^{3^-}\cdot \partial_3\boldsymbol{\varphi} + \underset{\approx}{\mathbf{B}}^{M^-}\cdot \underset{(s)}{g}{}^{P}_{M^-} N_P\mathbf{u} + \underset{\approx}{\mathbf{B}}^{3^-}\cdot \partial_3\mathbf{u} - \underset{\approx}{\mathbf{B}}\cdot\cdot\, \underset{\approx}{\mathbf{C}}\cdot\boldsymbol{\varphi} - \underset{\sim}{\boldsymbol{\beta}}\vartheta.
\end{aligned} \tag{3.31}$$

Definition 3.2. Relations obtained from (3.31) for $s=0$ are called the CE of the zeroth approximation, and those obtained for $s=1$ are called the CE of the first approximation.

It is easily seen that the CE of the zeroth approximation have the form

$$\begin{aligned}
\underset{\sim}{\mathbf{P}}_{(0)} &= \underset{\approx}{\mathbf{C}}^{M^-}\cdot N_P\mathbf{u} + \underset{\approx}{\mathbf{C}}^{3^-}\cdot \partial_3\mathbf{u} + \underset{\approx}{\mathbf{A}}^{M^-}\cdot N_P\boldsymbol{\varphi} + \underset{\approx}{\mathbf{A}}^{3^-}\cdot \partial_3\boldsymbol{\varphi} - \underset{\approx}{\mathbf{C}}\cdot\cdot\, \underset{\approx}{\mathbf{C}}\cdot\boldsymbol{\varphi} - \underset{\sim}{\mathbf{b}}\vartheta,\\
\underset{\sim}{\boldsymbol{\mu}}_{(0)} &= \underset{\approx}{\mathbf{D}}^{M^-}\cdot N_P\boldsymbol{\varphi} + \underset{\approx}{\mathbf{D}}^{3^-}\cdot \partial_3\boldsymbol{\varphi} + \underset{\approx}{\mathbf{B}}^{M^-}\cdot N_P\mathbf{u} + \underset{\approx}{\mathbf{B}}^{3^-}\cdot \partial_3\mathbf{u} - \underset{\approx}{\mathbf{B}}\cdot\cdot\, \underset{\approx}{\mathbf{C}}\cdot\boldsymbol{\varphi} - \underset{\sim}{\boldsymbol{\beta}}\vartheta,
\end{aligned} \tag{3.32}$$

and the CE of the first approximation can be written as

$$\begin{aligned}
\underset{\sim}{\mathbf{P}}_{(s)} &= \underset{\sim}{\mathbf{P}}_{(s-1)} + \underset{\approx}{\mathbf{C}}^{M^-}\cdot \underset{(s)}{A}{}^{P^-}_{M^+}(x^3)^s N_P\mathbf{u} + \underset{\approx}{\mathbf{A}}^{M^-}\cdot \underset{(s)}{A}{}^{P^-}_{M^+}(x^3)^s N_P\boldsymbol{\varphi},\\
\underset{\sim}{\boldsymbol{\mu}}_{(s)} &= \underset{\sim}{\boldsymbol{\mu}}_{(s-1)} + \underset{\approx}{\mathbf{D}}^{M^-}\cdot \underset{(s)}{A}{}^{P^-}_{M^+}(x^3)^s N_P\boldsymbol{\varphi} + \underset{\approx}{\mathbf{B}}^{M^-}\cdot \underset{(s)}{A}{}^{P^-}_{M^+}(x^3)^s N_P\mathbf{u}, \quad s\ge 1.
\end{aligned} \tag{3.33}$$

We find the corresponding expression for Fourier's thermal conduction law for the new parametrization of the domain occupied by the thin solid. Since Fourier's thermal conduction law [22, 25] has the form $\mathbf{q} = -\underset{\sim}{\boldsymbol{\Lambda}}\cdot\nabla T$, where the positive definite tensor of the second rank $\underset{\sim}{\boldsymbol{\Lambda}}$ is called the thermal conduction tensor, according to (3.5), Fourier's thermal conduction law of the sth approximation can be represented in the form

$$\mathbf{q}_{(s)} = -\boldsymbol{\Lambda}^{M^-}\cdot \underset{(s)}{g}{}^{P}_{M^-} N_P T - \boldsymbol{\Lambda}^{3^-}\partial_3 T, \quad \boldsymbol{\Lambda}^{m^-} = \underset{\sim}{\boldsymbol{\Lambda}}\cdot\mathbf{r}^{m^-}. \tag{3.34}$$

Hence, for Fourier's thermal conduction law of the zeroth approximation and the sth approximation (3.34), by analogy with (3.32) and (3.33), we have the expressions

$$\mathbf{q}_{(0)} = -\boldsymbol{\Lambda}^{M^-}\cdot N_P T - \boldsymbol{\Lambda}^{3^-}\partial_3 T, \quad \mathbf{q}_{(s)} = \mathbf{q}_{(s-1)} - \boldsymbol{\Lambda}^{M^-}\cdot \underset{(s)}{A}{}^{P}_{M^-} N_P T, \quad s\ge 1. \tag{3.35}$$

4. TO THE THEORY OF MOMENTS WITH RESPECT TO THE SYSTEM OF ORTHONORMAL CHEBYSHEV POLYNOMIALS OF THE SECOND KIND

We consider some tensor field $\mathbb{F}(x^1,x^2,x^3)$ depending on the coordinates x^1,x^2,x^3 of the domain occupied by a thin solid for its new parametrization [1, 2, 4]. By the thin solid we mean a three-dimensional solid one or two of whose dimensions is/are small compared with the other dimensions. In what follows, we mainly consider a thin solid one of whose dimensions is small compared with the other. For brevity, just as above, instead of $\mathbb{F}(x^1,x^2,x^3)$ we write $\mathbb{F}(x',x^3)$, where $x'=(x^1,x^2)$, $x^3\in[0,1]$. Moreover, we assume that the tensor fields under study are sufficiently smooth. For example, $\mathbb{F}(x',x^3)\in C_m(V\cup\partial V)$, $m\ge 1$, where V is the domain occupied by the thin solid and ∂V is its boundary. Then the tensor field $\mathbb{F}(x',x^3)$ with respect to the coordinate $x^3\in[0,1]$ for each fixed point $x'\in \overset{(-)}{S}$ (the interior base surface) can be expanded in a series in a system of shifted orthonormal Chebyshev polynomials of the second kind $\{\hat{U}^*_k\}_{k=0}^{\infty}$ [10]. This representation has the form

$$\mathbb{F}(x',x^3) = \sum_{k=0}^{\infty} \overset{(k)}{\mathbb{F}}(x')\hat{U}^*_k(x^3), \quad x'\in\overset{(-)}{S}, \quad x^3\in[0,1], \tag{4.1}$$

where $\overset{(k)}{\mathbb{F}}(x')$ is called the kth coefficient in the expansion of $\mathbb{F}(x', x^3)$ in a series in the system of polynomials $\{\hat{U}_k^*\}_{k=0}^{\infty}$.

Definition 4.1. An kth moment of some tensor field $\mathbb{F}(x', x^3)$ with respect to the system of polynomials $\{\hat{U}_k^*\}_{k=0}^{\infty}$, denoted by $\overset{(k)}{\mathbb{M}}(\mathbb{F})$, is defined to be the integral

$$\overset{(k)}{\mathbb{M}}(\mathbb{F}) = \int\limits_0^1 \mathbb{F}(x', x^3)\hat{U}_k^*(x^3)h^*(x^3)\,dx^3, \quad k \in \mathbb{N}_0. \tag{4.2}$$

The following assertion can easily be proved.

Proposition 4.1. *Any tensor fields $\mathbb{F}(x', x^3)$ and $\mathbb{G}(x', x^3)$ and any functions $\alpha(x')$ and $\beta(x')$ satisfy the relation*

$$\overset{(k)}{\mathbb{M}}[\alpha(x')\mathbb{F} + \beta(x')\mathbb{G}] = \alpha(x')\overset{(k)}{\mathbb{M}}(\mathbb{F}) + \beta(x')\overset{(k)}{\mathbb{M}}(\mathbb{G}), \quad k \in \mathbb{N}_0. \tag{4.3}$$

This implies that the moment operator is a linear operator.

Proposition 4.2. *The kth moment of the tensor field $\mathbb{F}(x', x^3)$ with respect to $\{\hat{U}_k^*\}_{k=0}^{\infty}$ is equal to the kth coefficient in the expansion of $\mathbb{F}(x', x^3)$ with respect to x^3 in this system of polynomials; i.e.,*

$$\overset{(k)}{\mathbb{M}}(\mathbb{F}) = \int\limits_0^1 \mathbb{F}(x', x^3)\hat{U}_k^*(x^3)h^*(x^3)\,dx^3 = \overset{(k)}{\mathbb{F}}(x'), \quad k \in \mathbb{N}_0. \tag{4.4}$$

The statement (4.3) follows from definition (4.2), and the statement (4.4) can be proved using (4.1), (4.2), and the fact that $\{\hat{U}_k^*\}_{k=0}^{\infty}$ is an orthonormal system.

It is easy to prove the following relations:

$$\overset{(k)}{\mathbb{M}}(\partial_i\mathbb{F}) = \begin{cases} \partial_I\overset{(k)}{\mathbb{F}}(x'), & i = I, \\ \overset{(k)}{\mathbb{F}}{}'(x'), & i = 3, \end{cases} \quad \overset{(k)}{\mathbb{M}}(\partial_i\partial_j\mathbb{F}) = \begin{cases} \partial_I\partial_J\overset{(k)}{\mathbb{F}}(x'), & i = I,\ j = J, \\ \partial_I\overset{(k)}{\mathbb{F}}{}'(x'), & i = I,\ j = 3, \\ \overset{(k)}{\mathbb{F}}{}''(x'), & i = j = 3, \end{cases} \tag{4.5}$$

$$\begin{aligned} \overset{(k)}{\mathbb{F}}{}'(x') &= 2^2(k+1)\sum_{p=0}^{\infty}\overset{(k+2p+1)}{\mathbb{F}}(x') = 2(k+1)\sum_{p=k}^{\infty}[1-(-1)^{k+p}]\overset{(p)}{\mathbb{F}}(x') \\ &= 2(k+1)\Big\{\sum_{p=k}^{N}[1-(-1)^{k+p}]\overset{(p)}{\mathbb{F}}(x') + \overset{(+)}{\mathbb{F}}{}'(x') - (-1)^k\overset{(-)}{\mathbb{F}}{}'(x')\Big\}, \\ \overset{(k)}{\mathbb{F}}{}''(x') &= (\overset{(k)}{\mathbb{F}}{}')' = 2^4(k+1)\sum_{p=0}^{\infty}(p+1)(k+p+2)\overset{(k+2p+2)}{\mathbb{F}}(x') \\ &= 2(k+1)\Big\{\sum_{p=k}^{N}(p-k)(k+p+2)[1+(-1)^{k+p}]\overset{(p)}{\mathbb{F}}(x') + \overset{(+)}{\mathbb{F}}{}''(x') + (-1)^k\overset{(-)}{\mathbb{F}}{}''(x')\Big\}, \end{aligned} \tag{4.6}$$

$$\begin{aligned} &\overset{(+)}{\mathbb{F}}{}'(x') = \sum_{p=N+1}^{\infty}\overset{(p)}{\mathbb{F}}(x'), \quad \overset{(-)}{\mathbb{F}}{}'(x') = \sum_{p=N+1}^{\infty}(-1)^p\overset{(p)}{\mathbb{F}}(x'), \\ &\overset{(+)}{\mathbb{F}}{}''(x') = \sum_{p=N+1}^{\infty}(p-k)(k+p+2)\overset{(p)}{\mathbb{F}}(x'), \quad \overset{(-)}{\mathbb{F}}{}''(x') = \sum_{p=N+1}^{\infty}(-1)^p(p-k)(k+p+2)\overset{(p)}{\mathbb{F}}(x'). \end{aligned} \tag{4.7}$$

We note that the first relation in (4.6) can serve as the "prime" operator, and the second relation can be obtained applying the "prime" operator twice to $\overset{(k)}{\mathbb{F}}$.

The following relations are generalizations of (4.5):

$$\overset{(k)}{\mathbb{M}}[P_N(x^3)\partial_i^p\partial_j^p\mathbb{F}] = \begin{cases} \partial_I^p\partial_J^q\overset{(k)}{\mathbb{M}}[P_N(x^3)\mathbb{F}], & i=I,\ j=J, \\ \partial_I\{\overset{(k)}{\mathbb{M}}[P_N(x^3)\mathbb{F}]\}^{(q)}, & i=I,\ j=3, \\ \{\overset{(k)}{\mathbb{M}}[P_N(x^3)\mathbb{F}]\}^{(p+q)}, & i=j=3, \end{cases} \tag{4.8}$$

where $P_N(x^3)$ is a polynomial of degree N, one has $k,\ N,\ p,\ q \in \mathbb{N}_0$, and $\{\overset{(k)}{\mathbb{M}}[P_N(x^3)\mathbb{F}]\}^{(m)}$, $m \in \mathbb{N}_0$, means that the "prime" operator is applied m times.

Definition (4.2) is used to prove the first rows in (4.5) and (4.8). The second and third rows in (4.5) are proved by using (2.5) and (2.8), respectively, while the second and third rows in (4.8) are proved by induction.

Using (2.1)–(2.3) and the last relation in (4.8), we can prove the following relations:

$$\begin{gathered} \overset{(n)}{\mathbb{M}}[(x^3)^s\partial_3^m\mathbb{F}] = \sum_{p=0}^{2s} 2^{-2s}\binom{2s}{p}\overset{(n-x+p)}{\mathbb{F}}{}^{(m)}(x'), \quad n-s\geq 0, \quad s,\ m\in\mathbb{N}_0, \\ \overset{(n)}{\mathbb{M}}[(x^3)^s\partial_3^m\mathbb{F}] = \sum_{p=1}^{2n+2} 2^{-2(n+1)}\binom{2n+2}{p}\overset{(p-1)}{\mathbb{F}}{}^{(m)}(x'), \quad s=n+1, \quad n,\ m\in\mathbb{N}_0, \\ \overset{(n)}{\mathbb{M}}[(x^3)^s\partial_3^m\mathbb{F}] = -\sum_{q=2}^{s-n} 2^{-2s}\binom{2s}{q-2}\overset{(s-n-q)}{\mathbb{F}}{}^{(m)}(x') + \sum_{p=s-n}^{2s} 2^{-2s}\binom{2s}{p}\overset{(n-s+p)}{\mathbb{F}}{}^{(m)}(x'), \\ s\geq n+2, \quad n,\ m\in\mathbb{N}_0. \end{gathered} \tag{4.9}$$

Based on (2.1)–(2.4) and (2.13), it is easy to verify the following relations:

$$\begin{gathered} \overset{(k)}{\mathbb{M}}[2^{2s}(x^3)^s fg] = \hat{U}_0^* \sum_{n=0}^{\infty}\sum_{q=0}^{2s}\sum_{p=0}^{k-s+q}\binom{2s}{q}\overset{(n+p)}{f}\overset{(n+k-s-p+q)}{g}, \quad k-s\geq 0, \quad s\geq 0, \\ \overset{(k)}{\mathbb{M}}[2^{2(k+1)}(x^3)^{k+1} fg] = \hat{U}_0^* \sum_{n=0}^{\infty}\sum_{q=1}^{2(k+1)}\sum_{p=0}^{q-1}\binom{2k+2}{q}\overset{(n+p)}{f}\overset{(n-p-1+q)}{g}, \quad k\geq 0, \\ \overset{(k)}{\mathbb{M}}[2^{2(k+s)}(x^3)^{k+s} fg] \\ = \hat{U}_0^* \sum_{n=0}^{\infty}\Big[-\sum_{q=2}^{s}\sum_{p=0}^{s-q}\binom{2(k+s)}{q-2}\overset{(n+p)}{f}\overset{(n+s-p-q)}{g} + \sum_{q=s}^{2(k+s)}\sum_{p=0}^{q-s}\binom{2(k+s)}{q}\overset{(n+p)}{f}\overset{(n-s-p+q)}{g}\Big], \\ s\geq 2, \quad k\geq 0. \end{gathered} \tag{4.10}$$

Here $f(x', x^3), g(x', x^3) \in C_m(V\cup\partial V)$, $m \geq 1$. It is clear that the first relation in (4.10) for $s=0$ implies the expression for the kth moment of the product of two functions.

Now we represent (4.9) for $m = 1$ in another form. Using simple transformations, the first relation in (4.6), and the first two relations in (4.7), from (4.9) for $m = 1$ we obtain

$$
\begin{aligned}
&\overset{(k)}{\mathbb{M}}[(x^3)^{s+1}\partial_3\mathbb{F}] = \overset{(k)}{\mathbb{M}}{}'[(x^3)^{s+1}\mathbb{F}] = \sum_{p=0}^{2s+2}\sum_{q=l-1}^{N} 2^{-(2s+1)}\binom{2s+2}{p} l[1+(-1)^{l+q}]\overset{(q)}{\mathbb{F}}(x') + (2k-s+1)\overset{(+)}{\mathbb{F}}{}',\\
&l \equiv k-s+p, \quad k \geq s+1, \quad N \geq k+s+1, \quad s \geq 0,\\
&\overset{(k)}{\mathbb{M}}[(x^3)^{k+1}\partial_3\mathbb{F}] = \overset{(k)}{\mathbb{M}}{}'[(x^3)^{k+1}\mathbb{F}] = \sum_{p=1}^{2k+2}\sum_{q=p-1}^{N} 2^{-(2k+1)}\binom{2k+2}{p} p[1+(-1)^{p+q}]\overset{(q)}{\mathbb{F}}(x') + (k+1)\overset{(+)}{\mathbb{F}}{}',\\
&N \geq 2k+1, \quad k \geq 0, \qquad\qquad (4.11)\\
&\overset{(k)}{\mathbb{M}}[(x^3)^{s+1}\partial_3\mathbb{F}] = \overset{(k)}{\mathbb{M}}{}'[(x^3)^{s+1}\mathbb{F}] = -\sum_{p=0}^{s-k-1}\sum_{q=p}^{N} 2^{-(2s+1)}\binom{2s+2}{s-k-1-p}(p+1)[1-(-1)^{p+q}]\overset{(q)}{\mathbb{F}}\\
&\qquad + \sum_{p=s+1-k}^{s+k+1}\sum_{q=p}^{N} 2^{-(2s+1)}\binom{2k+2}{s+1-k+p}(p+1)[1-(-1)^{p+q}]\overset{(q)}{\mathbb{F}} + a_{(s,k)}\overset{(-)}{\mathbb{F}}{}' + b_{(s,k)}\overset{(+)}{\mathbb{F}}{}',\\
&s \geq k+1, \quad N \geq s+k+1, \quad k \geq 0.
\end{aligned}
$$

Let us find the expression for $\overset{(k)}{\mathbb{M}}\big(\underset{(s)}{g}{}^{P}_{M^-} N_P\mathbb{F}\big)$. In view of (3.4) and (4.3), we obtain

$$
\overset{(k)}{\mathbb{M}}\big(\underset{(s)}{g}{}^{P}_{M^-} N_P\mathbb{F}\big) = \overset{(k)}{\mathbb{M}}\big(\underset{(s)}{g}{}^{P}_{M^-} \partial_P\mathbb{F}\big) - g^{3^-}_{P^+}\overset{(k)}{\mathbb{M}}\big(\underset{(s)}{g}{}^{P}_{M^-} \partial_3\mathbb{F}\big). \tag{4.12}
$$

Further, using (3.24), (4.3), (4.8), and (4.9) for $m = 1$, we find

$$
\begin{aligned}
\overset{(k)}{\mathbb{M}}\big(\underset{(s)}{g}{}^{P}_{M^-} \partial_P\mathbb{F}\big) &= \sum_{m=0}^{s}\underset{(m)}{A}{}^{P^-}_{M^+}\partial_P\overset{(k)}{\mathbb{M}}[(x^3)^m\mathbb{F}] = \sum_{m=0}^{k+1}\sum_{p=0}^{2m}\underset{(m)}{A}{}^{P^-}_{M^+}2^{-2m}\binom{2m}{p}\partial_P\overset{(k-m+p)}{\mathbb{F}}\\
&+\sum_{m=k+2}^{s}\underset{(m)}{A}{}^{P^-}_{M^+}\Bigg[-\sum_{p=2}^{m-k}2^{-2m}\binom{2m}{q-2}\partial_P\overset{(m-k-q)}{\mathbb{F}} + \sum_{p=m-k}^{2m}2^{-2m}\binom{2m}{p}\partial_P\overset{(k-m+p)}{\mathbb{F}}\Bigg], \quad k \geq 0, \quad s \geq 0.
\end{aligned}\tag{4.13}
$$

Hence for $s = 0$ and $s = 1$ we respectively obtain

$$
\begin{aligned}
&\overset{(k)}{\mathbb{M}}\big(\underset{(0)}{g}{}^{P}_{M^-} \partial_P\mathbb{F}\big) = \overset{(k)}{\mathbb{M}}(\partial_M\mathbb{F}), \quad k \geq 0,\\
&\overset{(k)}{\mathbb{M}}\big(\underset{(1)}{g}{}^{P}_{M^-} \partial_P\mathbb{F}\big) = \overset{(k)}{\mathbb{M}}\big[(g^{P^-}_{M^-} + x^3A^{P^-}_{M^+})\partial_P\mathbb{F}\big] = \partial_P\overset{(+)}{\mathbb{F}} + \frac{1}{4}A^{P^-}_{M^+}\partial_M\big(\overset{(k-1)}{\mathbb{F}} + 2\overset{(k)}{\mathbb{F}} + \overset{(k+1)}{\mathbb{F}}\big), \quad k \geq 0.
\end{aligned}\tag{4.14}
$$

Here we introduce the notation $A^{P^-}_{M^+} \equiv \underset{(1)}{A}{}^{P^-}_{M^+} = g^{P^-}_{M^-} - g^{P^-}_{M^+}$. Moreover, we assume that $\overset{(m)}{\mathbb{F}} = 0$ if $m < 0$. In what follows, we also assume that this condition is satisfied.

In a similar way, according to (3.24), (4.3), (4.8), and (4.11), we have

$$
\begin{aligned}
\overset{(k)}{\mathbb{M}}\big(x^3\underset{(s)}{g}{}^{P}_{M^-} \partial_3\mathbb{F}\big) &= \sum_{m=0}^{s}\underset{(m)}{A}{}^{P^-}_{M^+}\overset{(k)}{\mathbb{M}}{}'[(x^3)^{m+1}\mathbb{F}]\\
&= \sum_{m=0}^{k}\underset{(m)}{A}{}^{P^-}_{M^+}\sum_{p=0}^{2m+2}\sum_{q=l-1}^{N}2^{-(2m+1)}\binom{2m+2}{p} l[1+(-1)^{l+q}]\overset{(q)}{\mathbb{F}}(x')
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{m=0}^{s}\underset{(m)}{A}{}_{M^+}^{P^-}\Big\{-\sum_{p=0}^{m-k-1}\sum_{q=p}^{N}2^{-(2m+1)}\binom{2m+2}{m-k-1-p}(p+1)[1-(-1)^{p+q}]\overset{(q)}{\mathbb{F}}(x')\\
&+\sum_{p=m+1-k}^{m+k+1}\sum_{q=p}^{N}2^{-(2m+1)}\binom{2m+2}{m+1-k+p}(p+1)[1-(-1)^{p+q}]\overset{(q)}{\mathbb{F}}(x')\Big\}\\
&+\Big(\sum_{m=k+1}^{s}\underset{(m)}{A}{}_{M^+}^{P^-}a_{(s,k)}\Big)\overset{(-)}{\mathbb{F}}{}'+\Big[\sum_{m=0}^{k}(2k-m+1)\underset{(m)}{A}{}_{M^+}^{P^-}+\sum_{m=k+1}^{s}\underset{(m)}{A}{}_{M^+}^{P^-}b_{(s,k)}\Big]\overset{(+)}{\mathbb{F}}{}',
\end{aligned}
\tag{4.15}
$$

$$l\equiv k-m+p,\quad N\ge s+k+1,\quad k\ge 0,\quad s\ge 0.$$

Hence for $s=0$ and $s=1$ we respectively obtain

$$
\begin{aligned}
&\overset{(k)}{\mathbb{M}}\big(x^3\underset{(0)}{g}{}_{M^-}^{P}\partial_3\mathbb{F}\big)=g_{M^-}^{P^-}\overset{(k)}{\mathbb{M}}{}'(x^3\mathbb{F})=\frac{1}{4}g_{M^-}^{P^-}\big(\overset{(k-1)}{\mathbb{F}}+2\overset{(k)}{\mathbb{F}}+\overset{(k+1)}{\mathbb{F}}\big)\\
&\quad=g_{M^-}^{P^-}\Big[k\overset{(k)}{\mathbb{F}}+2(k+1)\Big(\sum_{p=k}^{N}\overset{(p)}{\mathbb{F}}-\overset{(k)}{\mathbb{F}}+\overset{(+)}{\mathbb{F}}{}'\Big)\Big],\quad k\ge 0,\\
&\overset{(k)}{\mathbb{M}}\big(x^3\underset{(1)}{g}{}_{M^-}^{P}\partial_3\mathbb{F}\big)=\overset{(k)}{\mathbb{M}}\big\{\big[g_{M^-}^{P^-}x^3+A_{M^+}^{P^-}(x^3)^2\big]_3\mathbb{F}\big\}=g_{M^-}^{P^-}\overset{(k)}{\mathbb{M}}{}'(x^3\mathbb{F})+A_{M^+}^{P^-}\overset{(k)}{\mathbb{M}}{}'[(x^3)^2\mathbb{F}]\\
&\quad=g_{M^-}^{P^-}\Big[k\overset{(k)}{\mathbb{F}}+2(k+1)\Big(\sum_{p=k}^{N}\overset{(p)}{\mathbb{F}}-\overset{(k)}{\mathbb{F}}+\overset{(+)}{\mathbb{F}}{}'\Big)\Big]\\
&\quad+\frac{1}{4}A_{M^+}^{P^-}\Big[(k-1)\overset{(k-1)}{\mathbb{F}}-4(k+2)\overset{(k)}{\mathbb{F}}-(k+3)\overset{(k+1)}{\mathbb{F}}+8(k+1)\Big(\sum_{p=k}^{N}\overset{(p)}{\mathbb{F}}+\overset{(+)}{\mathbb{F}}{}'\Big)\Big],\quad k\ge 0.
\end{aligned}
\tag{4.16}
$$

Taking into account (4.13) and (4.15), from (4.12) we obtain the desired relation in the form

$$
\begin{aligned}
&\overset{(k)}{\mathbb{M}}\big(\underset{(s)}{g}{}_{M^-}^{P}N_P\mathbb{F}\big)=\sum_{m=0}^{k+1}\sum_{p=0}^{2m}\underset{(m)}{A}{}_{M^+}^{P^-}2^{-2m}\binom{2m}{p}\partial_P\overset{(k-m+p)}{\mathbb{F}}\\
&\quad+\sum_{m=k+2}^{s}\underset{(m)}{A}{}_{M^+}^{P^-}\Big(-\sum_{p=2}^{m-k}2^{-2m}\binom{2m}{p-2}\partial_P\overset{(m-k-p)}{\mathbb{F}}+\sum_{p=m-k}^{2m}2^{-2m}\binom{2m}{p}\partial_P\overset{(k-m+p)}{\mathbb{F}}\Big)\\
&\quad-g_{P^+}^{3^-}\Big\{\sum_{m=0}^{k}\underset{(m)}{A}{}_{M^+}^{P^-}\sum_{p=0}^{2m+2}\sum_{q=l-1}^{N}2^{-(2m+1)}\binom{2m+2}{p}l[1+(-1)^{l+q}]\overset{(q)}{\mathbb{F}}(x')\\
&\quad+\sum_{m=k+1}^{s}\underset{(m)}{A}{}_{M^+}^{P^-}\Big[-\sum_{p=0}^{m-k-1}\sum_{q=p}^{N}2^{-(2m+1)}\binom{2m+2}{m-k-1-p}(p+1)[1-(-1)^{p+q}]\overset{(q)}{\mathbb{F}}(x')\\
&\quad+\sum_{p=m+1-k}^{m+k+1}\sum_{q=p}^{N}2^{-(2m+1)}\binom{2m+2}{m+1-k+p}(p+1)[1-(-1)^{p+q}]\overset{(q)}{\mathbb{F}}(x')\Big]\Big\},\\
&\quad-g_{P^+}^{3^-}\Big\{\Big(\sum_{m=k+1}^{s}\underset{(m)}{A}{}_{M^+}^{P^-}a_{(m,k)}\Big)\overset{(-)}{\mathbb{F}}{}'+\Big[\sum_{m=0}^{k}(2k-m+1)\underset{(m)}{A}{}_{M^+}^{P^-}+\sum_{m=k+1}^{s}\underset{(m)}{A}{}_{M^+}^{P^-}b_{(m,k)}\Big]\overset{(+)}{\mathbb{F}}{}'\Big\},
\end{aligned}
\tag{4.17}
$$

$$l\equiv k-m+p,\quad N\ge s+k+1,\quad k\ge 0,\quad s\ge 0.$$

Hence for $s = 0$ and $s = 1$ or in view of (4.14) and (4.16) we obtain

$$\begin{aligned}
&\overset{(k)}{\mathbb{M}}\big(\underset{(0)}{g}{}_{I^-}^{J} N_J\mathbb{F}\big) = \overset{(k)}{\mathbb{M}}(N_I\mathbb{F}) = \nabla_I\overset{(k)}{\mathbb{F}} - g_{I^+}^{3^-}\left[k\overset{(k)}{\mathbb{F}} + 2(k+1)\left(\sum_{p=k}^{N}\overset{(p)}{\mathbb{F}} - \overset{(k)}{\mathbb{F}} + \overset{(+)}{\mathbb{F}}{}'\right)\right], \quad k \geq 0,\\
&\overset{(k)}{\mathbb{M}}\big(\underset{(1)}{g}{}_{I^-}^{J} N_J\mathbb{F}\big) = \overset{(k)}{\mathbb{M}}\big[\big(g_{I^-}^{J^-} + x^3 A_{I^+}^{J^-}\big)N_P\mathbb{F}\big] = \nabla_I\overset{(k)}{\mathbb{F}} + \frac{1}{4}A_{I^+}^{J^-}\nabla_J\big(\overset{(k-1)}{\mathbb{F}} + 2\overset{(k)}{\mathbb{F}} + \overset{(k-2)}{\mathbb{F}}\big)\\
&\qquad - g_{J^+}^{3^-}\left\{g_{I^-}^{J^-}\left[k\overset{(k)}{\mathbb{F}} + 2(k+1)\left(\sum_{p=k}^{N}\overset{(p)}{\mathbb{F}} - \overset{(k)}{\mathbb{F}} + \overset{(+)}{\mathbb{F}}{}'\right)\right]\right.\\
&\qquad \left. + \frac{1}{4}A_{I^+}^{J^-}\left[(k-1)\overset{(k-1)}{\mathbb{F}} - 4(k+2)\overset{(k)}{\mathbb{F}} - (k+3)\overset{(k+1)}{\mathbb{F}} + 8(k+1)\left(\sum_{p=k}^{N}\overset{(p)}{\mathbb{F}} + \overset{(+)}{\mathbb{F}}{}'\right)\right]\right\}, \quad k \geq 0.
\end{aligned} \tag{4.18}$$

5. SYSTEMS OF EQUATIONS OF MOTION, EQUATIONS OF HEAT INFLUX, AND CONSTITUTIVE EQUATIONS IN MOMENTS OF THE MOMENT MTDS

The above approximate equations (3.23) and (3.27) and the CE (3.31) and (3.34) contain terms with the factor $\underset{(n)}{g}{}_{I^-}^{J} N_J\mathbb{F}$. Moreover, each of relations (3.31) contains two similar terms, while the other relations contain only one such term. Therefore, on the basis of (4.17), these relations permit obtaining systems of the corresponding equations and the CE in moments of an arbitrary approximation. They are as cumbersome as (4.17). Hence we do not write them out in general form but obtain systems of equations and the CE in terms of moments of the zeroth and first approximations.

We note that it is rather difficult to derive similar general relations using the system of Legendre polynomials.

5.1. Systems of equations of motion and heat influx in moments of the zeroth and first approximations of the moment MTDS

The equations of motion of the zeroth approximation of the moment MTDS are obtained from (3.23) for $r = 0$ and have the form

$$N_I\mathbf{P}^{I^-} + \partial_3\mathbf{P}^{3^-} + \rho\mathbf{F} = \rho\partial_t^2\mathbf{u}, \quad N_I\boldsymbol{\mu}^{I^-} + \partial_3\boldsymbol{\mu}^{3^-} + \underset{\approx}{\mathbf{C}} \cdot\cdot \underset{\sim}{\mathbf{P}}^T + \rho\mathbf{m} = \underset{\sim}{\mathbf{J}} \cdot \partial_t^2\boldsymbol{\varphi}. \tag{5.1}$$

In what follows, we consider only materials homogeneous in x^3 unless otherwise specified.

Applying the kth moment operator to Eqs. (5.1), in view of (4.3) and the first relations in (4.5), (4.6), and (4.18), we obtain the following system of equations of motion in moments of the zeroth approximation of the moment MTDS:

$$\begin{aligned}
&\overset{(k)}{\mathbf{M}}(N_I\mathbf{P}^{I^-}) + \overset{(k)}{\mathbf{M}}(\partial_3\mathbf{P}^{3^-}) + \rho\overset{(k)}{\mathbf{F}} = \rho\partial_t^2\overset{(k)}{\mathbf{u}},\\
&\overset{(k)}{\mathbf{M}}(N_I\boldsymbol{\mu}^{I^-}) + \overset{(k)}{\mathbf{M}}(\partial_3\boldsymbol{\mu}^{3^-}) + \underset{\approx}{\mathbf{C}} \cdot\cdot \overset{(k)}{\underset{\sim}{\mathbf{P}}}{}^T + \rho\overset{(k)}{\mathbf{m}} = \underset{\sim}{\mathbf{J}} \cdot \partial_t^2\overset{(k)}{\boldsymbol{\varphi}}, \quad k \in \mathbb{N}_0,
\end{aligned} \tag{5.2}$$

where the sums of the first two terms on the left-hand side in(5.2) have the form

$$\begin{aligned}
&\overset{(k)}{\mathbf{M}}(N_I\mathbf{P}^{I^-}) + \overset{(k)}{\mathbf{M}}(\partial_3\mathbf{P}^{3^-}) = \nabla_I\overset{(k)}{\mathbf{P}}{}^{I^-} - g_{I^+}^{3^-}\left[k\overset{(k)}{\mathbf{P}}{}^{I^-} + 2(k+1)\left(\sum_{p=k}^{N}\overset{(p)}{\mathbf{P}}{}^{I^-} - \overset{(k)}{\mathbf{P}}{}^{I^-}\right)\right]\\
&\qquad +2(k+1)\sum_{p=k}^{N}[1-(-1)^{k+p}]\overset{(k)}{\mathbf{P}}{}^{3^-} + 2(k+1)\left[\left(\mathbf{r}^{3^-} - g_{I^+}^{3^-}\mathbf{r}^{I^-}\right)\cdot\overset{(+)}{\underset{\sim}{\mathbf{P}}}{}' - (-1)^k\mathbf{r}^{3^-}\cdot\overset{(-)}{\underset{\sim}{\mathbf{P}}}{}'\right]; \quad \mathbf{P} \to \boldsymbol{\mu}.
\end{aligned} \tag{5.3}$$

Here the notation $\mathbf{P} \to \boldsymbol{\mu}$ means that this relation should be supplemented with the formula obtained from this relation by replacing the letter $\mathbf{P}$ by $\boldsymbol{\mu}$. This notation will be used in the following.

In (5.3), by analogy with the first two relations in (4.7), we introduce the notation

$$\overset{(+)}{\underset{\sim}{\mathbf{P}}}{}' = \mathbf{r}_{m^-} \overset{(+)}{\underset{\sim}{\mathbf{P}}}{}'^{m^-} = \sum_{p=N+1}^{\infty} \overset{(p)}{\underset{\sim}{\mathbf{P}}}, \quad \overset{(-)}{\underset{\sim}{\mathbf{P}}}{}' = \mathbf{r}_{m^-} \overset{(-)}{\underset{\sim}{\mathbf{P}}}{}'^{m^-} = \sum_{p=N+1}^{\infty} (-1)^p \overset{(p)}{\underset{\sim}{\mathbf{P}}}; \quad \mathbf{P} \to \boldsymbol{\mu}. \tag{5.4}$$

The equations of motion of the first approximation of the moment MTDS are found from (3.23) for $r = 1$. They have the form

$$\underset{(1)}{g}{}^{I}_{J^-} N_I \mathbf{P}^{J^-} + \partial_3 \mathbf{P}^{3^-} + \rho \mathbf{F} = \rho \partial_t^2 \mathbf{u}, \quad \underset{(1)}{g}{}^{I}_{J^-} N_I \boldsymbol{\mu}^{J^-} + \partial_3 \boldsymbol{\mu}^{3^-} + \underset{\approx}{\mathbf{C}} \cdot\cdot \underset{\sim}{\mathbf{P}}^T + \rho \mathbf{m} = \underset{\sim}{\mathbf{J}} \cdot \partial_t^2 \boldsymbol{\varphi}. \tag{5.5}$$

Applying the kth moment operator to Eqs. (5.5) and using (4.3), the first relations in (4.5) and (4.6) and the second relation in (4.18), we obtain the following system of equations of motion in moments of the first approximation of the moment MTDS:

$$\begin{gathered}
\overset{(k)}{\mathbf{M}}\big(\underset{(1)}{g}{}^{I}_{J^-} N_I \mathbf{P}^{J^-}\big) + \overset{(k)}{\mathbf{M}}(\partial_3 \mathbf{P}^{3^-}) + \rho \overset{(k)}{\mathbf{F}} = \rho \partial_t^2 \overset{(k)}{\mathbf{u}}, \\
\overset{(k)}{\mathbf{M}}\big(\underset{(1)}{g}{}^{I}_{J^-} N_I \boldsymbol{\mu}^{J^-}\big) + \overset{(k)}{\mathbf{M}}(\partial_3 \boldsymbol{\mu}^{3^-}) + \underset{\approx}{\mathbf{C}} \cdot\cdot \overset{(k)}{\underset{\sim}{\mathbf{P}}}{}^T + \rho \overset{(k)}{\mathbf{m}} = \underset{\sim}{\mathbf{J}} \cdot \partial_t^2 \overset{(k)}{\boldsymbol{\varphi}}, \quad k \in \mathbb{N}_0, \\
\big(\overset{(n)}{\underset{\sim}{\mathbf{P}}} = 0 \ \big(\overset{(n)}{\underset{\sim}{\mathbf{P}}}{}^{m^-} = 0\big), \ \overset{(n)}{\underset{\sim}{\boldsymbol{\mu}}} = 0 \ \big(\overset{(n)}{\underset{\sim}{\boldsymbol{\mu}}}{}^{m^-} = 0\big) \ \text{for} \ n \le 0\big),
\end{gathered} \tag{5.6}$$

where the expression for the sum of the first two terms on the left-hand side in the first relation in (5.6) has the form

$$\begin{aligned}
&\overset{(k)}{\mathbf{M}}\big(\underset{(1)}{g}{}^{I}_{J^-} N_I \mathbf{P}^{J^-}\big) + \overset{(k)}{\mathbf{M}}(\partial_3 \mathbf{P}^{3^-}) = \nabla_I \overset{(k)}{\mathbf{P}}{}^{I^-} + \frac{1}{4} A^{J^-}_{I^+} \nabla_J \big(\overset{(k-1)}{\mathbf{P}}{}^{I^-} + 2\overset{(k)}{\mathbf{P}}{}^{I^-} + \overset{(k+1)}{\mathbf{P}}{}^{I^-}\big) \\
&\quad - g^{3^-}_{J^+} \Bigg\{ g^{J^-}_{I^-} \Bigg[k\overset{(k)}{\mathbf{P}}{}^{I^-} + 2(k+1)\Bigg(\sum_{p=k}^{N} \overset{(p)}{\mathbf{P}}{}^{I^-} - \overset{(k)}{\mathbf{P}}{}^{I^-}\Bigg)\Bigg] + \frac{1}{4} A^{J^-}_{I^+} \Bigg[(k-1)\overset{(k-1)}{\mathbf{P}}{}^{I^-} - 4(k+2)\overset{(k)}{\mathbf{P}}{}^{I^-} \\
&\quad - (k+3)\overset{(k+1)}{\mathbf{P}}{}^{I^-} + 8(k+1)\sum_{p=k}^{N} \overset{(p)}{\mathbf{P}}{}^{I^-} \Bigg]\Bigg\} + 2(k+1)\sum_{p=k}^{N} [1 - (-1)^{k+p}] \overset{(p)}{\mathbf{P}}{}^{3^-} \\
&\quad + 2(k+1)\Big\{ \big[\mathbf{r}^{3^-} - g^{3^-}_{I^+}\big(g^{I^-}_{J^+} + A^{I^-}_{J^+}\big)\mathbf{r}^{J^-}\big] \cdot \overset{(+)}{\underset{\sim}{\mathbf{P}}}{}' - (-1)^k \mathbf{r}^{3^-} \cdot \overset{(-)}{\underset{\sim}{\mathbf{P}}}{}' \Big\}; \quad \mathbf{P} \to \boldsymbol{\mu}.
\end{aligned} \tag{5.7}$$

By analogy with (5.1) and (5.5), from (3.27) for $r = 0$ and $r = 1$ we obtain the equations of heat influx in the zeroth and first approximations of the moment MTDS. These equations have the form

$$\begin{gathered}
-N_I q^{I^-} - \partial_3 q^{3^-} + \rho q - T_0 \frac{d}{dt}(\underset{\sim}{\mathbf{a}} \cdot\cdot \underset{\sim}{\mathbf{P}}^T + \underset{\sim}{\mathbf{d}} \cdot\cdot \boldsymbol{\mu}^T) + W^* = \rho c_p \partial_t T, \\
-\underset{(1)}{g}{}^{P}_{M^-} N_P q^{M^-} - \partial_3 q^{3^-} + \rho q - T_0 \frac{d}{dt}(\underset{\sim}{\mathbf{a}} \cdot\cdot \underset{\sim}{\mathbf{P}}^T + \underset{\sim}{\mathbf{d}} \cdot\cdot \boldsymbol{\mu}^T) + W^* = \rho c_p \partial_t T.
\end{gathered}$$

Applying the kth moment operator to these equations and using (4.3) and the first relations in (4.5) and (4.6) and (4.8), by analogy with (5.2) and (5.6), we obtain systems of equations of heat influx in moments of the zeroth and first approximations of the moment MTDS:

$$-\Big[\overset{(k)}{M}(N_I q^{I^-}) + \overset{(k)}{M}(\partial_3 q^{3^-})\Big] + \rho \overset{(k)}{q} - T_0 \frac{d}{dt}(\underset{\sim}{\mathbf{a}} \cdot\cdot \overset{(k)}{\underset{\sim}{\mathbf{P}}}{}^T + \underset{\sim}{\mathbf{d}} \cdot\cdot \overset{(k)}{\underset{\sim}{\boldsymbol{\mu}}}{}^T) + \overset{(k)}{W}{}^* = \rho c_p \partial_t \overset{(k)}{T}, \quad k \in \mathbb{N}_0, \tag{5.8}$$

$$-\Big[\overset{(k)}{M}\big(\underset{(1)}{g}{}^{I}_{J^-} N_I q^{J^-}\big) + \overset{(k)}{M}\big(\partial_3 q^{3^-}\big)\Big] + \rho \overset{(k)}{q} - T_0 \frac{d}{dt}(\underset{\sim}{\mathbf{a}} \cdot\cdot \overset{(k)}{\underset{\sim}{\mathbf{P}}}{}^T + \underset{\sim}{\mathbf{d}} \cdot\cdot \overset{(k)}{\underset{\sim}{\boldsymbol{\mu}}}{}^T) + \overset{(k)}{W}{}^* = \rho c_p \partial_t \overset{(k)}{T}, \quad k \in \mathbb{N}_0, \tag{5.9}$$

$$\big(\overset{(k)}{\mathbf{q}} = 0 \ \big(\overset{(k)}{q}{}^{m^-} = 0\big) \ \text{for} \ n \le 0\big).$$

We note that, the sums of two terms in square brackets on the left-hand sides of Eqs. (5.8) and (5.9) are obtained from (5.3) and (5.7), respectively, if the letter P in them is replaced by the letter q. Here, by analogy with (5.4) we have introduced the notation

$$\overset{(+)}{\mathbf{q}}{}' = \overset{(+)}{q}{}'^{\,m^-}\mathbf{r}_{m^-} = \sum_{p=N+1}^{\infty} \overset{(p)}{q}, \quad \overset{(-)}{\mathbf{q}}{}' = \overset{(-)}{q}{}'^{\,m^-}\mathbf{r}_{m^-} = \sum_{p=N+1}^{\infty} (-1)^p \overset{(p)}{q}.$$

The systems of equations of motion (5.2) and (5.6) and of heat influx (5.8) and (5.9) in moments of the zeroth and first approximations of the moment MTDS are infinite systems of equations. In this case, each equation in these systems contains infinitely many terms (a similar picture also takes place for the system of equations in moments of arbitrary approximation). Therefore, they should be reduced to finite systems where each equation contains finitely many terms. This reduction is performed as follows: we fix a nonnegative integer N and, instead of a given infinite system, consider the system consisting only of the first $N+1$ equations. Each of these equations contains moments of the desired variables whose maximum order does not exceed N. In other words, in each equation of the system in question, we neglect the moments of the desired variables whose order is greater than N. In this connection, we introduce the following definition.

Definition 5.1. The system of equations in moments which consists of the first $N+1$ equations of the corresponding infinite system of equations of motion (heat influx) of the kth approximation and such that each of its equations does not contain any moments of the desired variables whose order is greater than N is called the system of equations of motion (heat influx) in moments of the (r, N)th approximation of the moment MTDS.

By this definition, for example, the systems of equations of motion and heat influx in moments of the $(0, N)$th approximation of the moment MTDS derived from (5.2) and (5.8) have the form

$$\begin{gathered}
\nabla_I \overset{(k)}{\underset{\sim}{\mathbf{P}}}{}^{I^-} - g_{I^+}^{3^-}\left[k\overset{(k)}{\underset{\sim}{\mathbf{P}}}{}^{I^-} + 2(k+1)\left(\sum_{p=k}^{N}\overset{(p)}{\underset{\sim}{\mathbf{P}}}{}^{I^-} - \overset{(k)}{\underset{\sim}{\mathbf{P}}}{}^{I^-}\right)\right] + 2(k+1)\sum_{p=k}^{N}[1-(-1)^{k+p}]\overset{(p)}{\underset{\sim}{\mathbf{P}}}{}^{3^-} + \rho\overset{(k)}{\mathbf{F}} = \rho\partial_t^2\overset{(k)}{\mathbf{u}}, \\
\nabla_I \overset{(k)}{\underset{\sim}{\boldsymbol{\mu}}}{}^{I^-} - g_{I^+}^{3^-}\left[k\overset{(k)}{\underset{\sim}{\boldsymbol{\mu}}}{}^{I^-} + 2(k+1)\left(\sum_{p=k}^{N}\overset{(p)}{\underset{\sim}{\boldsymbol{\mu}}}{}^{I^-} - \overset{(k)}{\underset{\sim}{\boldsymbol{\mu}}}{}^{I^-}\right)\right] + 2(k+1)\sum_{p=k}^{N}[1-(-1)^{k+p}]\overset{(p)}{\underset{\sim}{\boldsymbol{\mu}}}{}^{3^-} \\
+ \underset{\approx}{\mathbf{C}} \cdot\cdot \overset{(k)}{\underset{\sim}{\mathbf{P}}}{}^{T} + \rho\overset{(k)}{\mathbf{m}} = \underset{\sim}{\mathbf{J}} \cdot \partial_t^2 \overset{(k)}{\boldsymbol{\varphi}} \quad (k = \overline{0,\, N}),
\end{gathered} \tag{5.10}$$

$$\begin{gathered}
-\nabla_I \overset{(k)}{q}{}^{I^-} + g_{I^+}^{3^-}\left[k\overset{(k)}{q}{}^{I^-} + 2(k+1)\left(\sum_{p=k}^{N}\overset{(p)}{q}{}^{I^-} - \overset{(k)}{q}{}^{I^-}\right)\right] - 2(k+1)\sum_{p=k}^{N}[1-(-1)^{k+p}]\overset{(p)}{q}{}^{3^-} \\
+\rho\overset{(k)}{q} - T_0\frac{d}{dt}(\underset{\sim}{\mathbf{a}} \cdot\cdot \overset{(k)}{\underset{\sim}{\mathbf{P}}}{}^{T} + \underset{\sim}{\mathbf{d}} \cdot\cdot \overset{(k)}{\underset{\sim}{\boldsymbol{\mu}}}{}^{T}) + \overset{(k)}{W}{}^{*} = \rho c_p \partial_t \overset{(k)}{T} \quad (k = \overline{0,\, N}),
\end{gathered} \tag{5.11}$$

where the notation $k = \overline{0,\, N}$ means that k ranges from 0 to N.

In (4.17), we neglect the moments whose order is greater than N, replace the index s by r, and obtain the following relation in the (r, N)th approximation:

$$\begin{aligned}
\underset{(r)}{\overset{(k)}{\mathbb{M}}}\big(g_{M^-}^{P} N_P \mathbb{F}\big) &= \sum_{m=0}^{k+1}\sum_{p=0}^{2m} \underset{(m)}{A}{}_{M^+}^{P^-} 2^{-2m}\binom{2m}{p}\partial_P \overset{(k-m+p)}{\mathbb{F}} \\
&+ \sum_{m=k+2}^{r} \underset{(m)}{A}{}_{M^+}^{P^-}\left(-\sum_{p=2}^{m-k} 2^{-2m}\binom{2m}{p-2}\partial_P \overset{(m-k-p)}{\mathbb{F}} + \sum_{p=m-k}^{2m} 2^{-2m}\binom{2m}{p}\partial_P \overset{(k-m+p)}{\mathbb{F}}\right) \\
&- g_{P^+}^{3^-}\left\{\sum_{m=0}^{k} \underset{(m)}{A}{}_{M^+}^{P^-} \sum_{p=0}^{2m+2}\sum_{q=l-1}^{N} 2^{-(2m+1)}\binom{2m+2}{p} l[1+(-1)^{l+q}]\overset{(q)}{\mathbb{F}}(x')\right. \\
&+ \sum_{m=k+1}^{r} \underset{(m)}{A}{}_{M^+}^{P^-}\left[-\sum_{p=0}^{m-k-1}\sum_{q=p}^{N} 2^{-(2m+1)}\binom{2m+2}{m-k-1-p}(p+1)[1-(-1)^{p+q}]\overset{(q)}{\mathbb{F}}(x')\right.
\end{aligned}$$

$$+\sum_{p=m+1-k}^{m+k+1}\sum_{q=p}^{N}2^{-(2m+1)}\binom{2m+2}{m+1-k+p}(p+1)[1-(-1)^{p+q}]\overset{(q)}{\mathbb{F}}(x')\Bigg]\Bigg\},$$
$$l\equiv k-m+p,\quad N\geq r+k+1,\quad k\geq 0,\quad r\geq 0,$$

which easily permits us to write out the systems of equations of motion and heat influx in moments of the (r, N)th approximation. Since these systems of equations are very cumbersome (in what follows, we assume that they are known), we do not present them here.

5.2. Systems of Hooke's laws and Fourier's thermal conduction laws in moments of the zeroth and first approximations of the moment MTDS

By (4.3), the first relations in (4.5), (4.6), and (4.18), from (3.32) we obtain the following system of Hooke's laws in moments of the zeroth approximation:

$$\begin{aligned}
&\underset{\sim}{\overset{(k)}{\mathbf{P}}}_{(0)}=\underset{\sim}{\overset{(k)}{\mathbf{P}}}_{(0,N)}+\underset{\simeq}{\mathbf{C}}^{3-}_{(0,k)}\cdot\overset{(+)}{\mathbf{u}}{}'+\underset{\simeq}{\mathbf{C}}^{3-}_{(k)}\cdot\overset{(-)}{\mathbf{u}}{}'+\underset{\simeq}{\mathbf{A}}^{3-}_{(0,k)}\cdot\overset{(+)}{\boldsymbol{\varphi}}{}'+\underset{\simeq}{\mathbf{A}}^{3-}_{(k)}\cdot\overset{(-)}{\boldsymbol{\varphi}}{}',\\
&\underset{\sim}{\overset{(k)}{\boldsymbol{\mu}}}_{(0)}=\underset{\sim}{\overset{(k)}{\boldsymbol{\mu}}}_{(0,N)}+\underset{\simeq}{\mathbf{D}}^{3-}_{(0,k)}\cdot\overset{(+)}{\boldsymbol{\varphi}}{}'+\underset{\simeq}{\mathbf{D}}^{3-}_{(k)}\cdot\overset{(-)}{\boldsymbol{\varphi}}{}'+\underset{\simeq}{\mathbf{B}}^{3-}_{(0,k)}\cdot\overset{(+)}{\mathbf{u}}{}'+\underset{\simeq}{\mathbf{B}}^{3-}_{(k)}\cdot\overset{(-)}{\mathbf{u}}{}',\quad k\in\mathbb{N}_0,
\end{aligned}\tag{5.12}$$

$$\begin{aligned}
\underset{\sim}{\overset{(k)}{\mathbf{P}}}_{(0,N)}&=\underset{\simeq}{\mathbf{C}}^{M-}\cdot\left\{\nabla_M\overset{(k)}{\mathbf{u}}(x')-g^{3-}_{M^+}\left[k\overset{(k)}{\mathbf{u}}(x')+2(k+1)\left(\sum_{p=k}^{N}\overset{(p)}{\mathbf{u}}(x')-\overset{(k)}{\mathbf{u}}(x')\right)\right]\right\}\\
&+2(k+1)\underset{\simeq}{\mathbf{C}}^{3-}\cdot\sum_{p=k}^{N}[1-(-1)^{k+p}]\overset{(p)}{\mathbf{u}}(x')\\
&+\underset{\simeq}{\mathbf{A}}^{M-}\cdot\left\{\nabla_M\overset{(k)}{\boldsymbol{\varphi}}(x')-g^{3-}_{M^+}\left[k\overset{(k)}{\boldsymbol{\varphi}}(x')2(k+1)\left(\sum_{p=k}^{N}\overset{(p)}{\boldsymbol{\varphi}}(x')-\overset{(k)}{\boldsymbol{\varphi}}(x')\right)\right]\right\}\\
&+2(k+1)\underset{\simeq}{\mathbf{A}}^{3-}\cdot\sum_{p=k}^{N}[1-(-1)^{k+p}]\overset{(p)}{\boldsymbol{\varphi}}(x')-\underset{\approx}{\mathbf{C}}\cdot\cdot\,\underset{\simeq}{\mathbf{C}}\cdot\overset{(k)}{\boldsymbol{\varphi}}(x')-\underset{\sim}{\mathbf{b}}\overset{(k)}{\vartheta},\\
\underset{\sim}{\overset{(k)}{\boldsymbol{\mu}}}_{(0,N)}&=\underset{\simeq}{\mathbf{D}}^{M-}\cdot\left\{\nabla_M\overset{(k)}{\boldsymbol{\varphi}}(x')-g^{3-}_{M^+}\left[k\overset{(k)}{\boldsymbol{\varphi}}(x')+2(k+1)\left(\sum_{p=k}^{N}\overset{(p)}{\boldsymbol{\varphi}}(x')-\overset{(k)}{\boldsymbol{\varphi}}(x')\right)\right]\right\}\\
&+2(k+1)\underset{\simeq}{\mathbf{D}}^{3-}\cdot\sum_{p=k}^{N}[1-(-1)^{k+p}]\overset{(p)}{\boldsymbol{\varphi}}(x')\\
&+\underset{\simeq}{\mathbf{B}}^{M-}\cdot\left\{\nabla_M\overset{(k)}{\mathbf{u}}(x')-g^{3-}_{M^+}\left[k\overset{(k)}{\mathbf{u}}(x')+2(k+1)\left(\sum_{p=k}^{N}\overset{(p)}{\mathbf{u}}(x')-\overset{(k)}{\mathbf{u}}(x')\right)\right]\right\}\\
&+2(k+1)\underset{\simeq}{\mathbf{B}}^{3-}\cdot\sum_{p=k}^{N}[1-(-1)^{k+p}]\overset{(p)}{\mathbf{u}}(x')-\underset{\approx}{\mathbf{B}}\cdot\cdot\,\underset{\simeq}{\mathbf{C}}\cdot\overset{(k)}{\boldsymbol{\varphi}}(x')-\underset{\sim}{\boldsymbol{\beta}}\overset{(k)}{\vartheta}(x'),\quad k\in\mathbb{N}_0.
\end{aligned}\tag{5.13}$$

Here we have introduced the notation

$$\underset{\simeq}{\mathbf{C}}^{3-}_{(0,k)}=2(k+1)\underset{\approx}{\mathbf{C}}\cdot\left(\mathbf{r}^{3-}-g^{3-}_{M^+}\mathbf{r}^M\right),\quad \underset{\simeq}{\mathbf{C}}^{3-}_{(k)}=2(k+1)(-1)^{k+1}\underset{\approx}{\mathbf{C}}\cdot\mathbf{r}^{3-},\quad k\in\mathbb{N}_0.\tag{5.14}$$

Similar relations for $\underset{\simeq}{\mathbf{A}}^{3-}_{(0,k)}$, $\underset{\simeq}{\mathbf{A}}^{3-}_{(k)}$, $\underset{\simeq}{\mathbf{D}}^{3-}_{(0,k)}$, $\underset{\simeq}{\mathbf{D}}^{3-}_{(k)}$, and $\underset{\simeq}{\mathbf{B}}^{3-}_{(0,k)}$, $\underset{\simeq}{\mathbf{B}}^{3-}_{(k)}$ can be obtained from (5.14) if the letter $\mathbf{C}$ is replaced by $\mathbf{A}$, $\mathbf{D}$, and $\mathbf{B}$, respectively.

Moreover, by analogy with the first two relations in (4.7), we have

$$\overset{(+)}{\mathbf{u}}{}'(x')=\sum_{p=N+1}^{\infty}\overset{(p)}{\mathbf{u}}(x'),\quad \overset{(-)}{\mathbf{u}}{}'(x')=\sum_{p=N+1}^{\infty}(-1)^p\overset{(p)}{\mathbf{u}}(x'),\quad \mathbf{u}\to\boldsymbol{\varphi}.$$

Further, by applying the kth moment operator to the CE obtained from (3.31) for $s = 1$ and by taking into account (4.3), the first relations in (4.5) and (4.6), and the second relation in (4.18), we obtain the following system of Hooke's laws in moments of the first approximation:

$$\begin{aligned}
&\underset{\sim}{\overset{(k)}{\mathbf{P}}}_{(1)} = \underset{\sim}{\overset{(k)}{\mathbf{P}}}_{(1,N)} + \underset{\approx}{\mathbf{C}}^{3^-}_{(1,k)} \cdot \overset{(+)}{\mathbf{u}}{}' + \underset{\approx}{\mathbf{C}}^{3^-}_{(k)} \cdot \overset{(-)}{\mathbf{u}}{}' + \underset{\approx}{\mathbf{A}}^{3^-}_{(1,k)} \cdot \overset{(+)}{\boldsymbol{\varphi}}{}' + \underset{\approx}{\mathbf{A}}^{3^-}_{(k)} \cdot \overset{(-)}{\boldsymbol{\varphi}}{}',\\
&\underset{\sim}{\overset{(k)}{\boldsymbol{\mu}}}_{(1)} = \underset{\sim}{\overset{(k)}{\boldsymbol{\mu}}}_{(1,N)} + \underset{\approx}{\mathbf{D}}^{3^-}_{(1,k)} \cdot \overset{(+)}{\boldsymbol{\varphi}}{}' + \underset{\approx}{\mathbf{D}}^{3^-}_{(k)} \cdot \overset{(-)}{\boldsymbol{\varphi}}{}' + \underset{\approx}{\mathbf{B}}^{3^-}_{(1,k)} \cdot \overset{(+)}{\mathbf{u}}{}' + \underset{\approx}{\mathbf{B}}^{3^-}_{(k)} \cdot \overset{(-)}{\mathbf{u}}{}', \quad k \in \mathbb{N}_0,
\end{aligned} \tag{5.15}$$

$$\begin{aligned}
&\underset{\sim}{\overset{(k)}{\mathbf{P}}}_{(1,N)} = \underset{\sim}{\overset{(k)}{\mathbf{P}}}_{(0,N)} + \frac{1}{4} A^{P^-}_{M^+} \underset{\approx}{\mathbf{C}}^{M^-} \cdot \Big\{ \nabla_P \big(\overset{(k-1)}{\mathbf{u}} + 2\overset{(k)}{\mathbf{u}} + \overset{(k+1)}{\mathbf{u}}\big) - g^{3^-}_{P^+} \Big[(k-1)\overset{(k-1)}{\mathbf{u}} - 4(k+2)\overset{(k)}{\mathbf{u}}\\
&\qquad - (k+3)\overset{(k+1)}{\mathbf{u}} + 8(k+1) \sum_{p=k}^{N} \overset{(p)}{\mathbf{u}} \Big] \Big\} + \frac{1}{4} A^{P^-}_{M^+} \underset{\approx}{\mathbf{A}}^{M^-} \cdot \Big\{ \nabla_P \big(\overset{(k-1)}{\boldsymbol{\varphi}} + 2\overset{(k)}{\boldsymbol{\varphi}} + \overset{(k+1)}{\boldsymbol{\varphi}}\big)\\
&\qquad - g^{3^-}_{P^+} \Big[(k-1)\overset{(k-1)}{\boldsymbol{\varphi}} - 4(k+2)\overset{(k)}{\boldsymbol{\varphi}} - (k+3)\overset{(k+1)}{\boldsymbol{\varphi}} + 8(k+1) \sum_{p=k}^{N} \overset{(p)}{\boldsymbol{\varphi}} \Big] \Big\};\\
&\mathbf{P} \to \boldsymbol{\mu}, \quad \mathbf{C} \to \mathbf{D}, \quad \mathbf{A} \to \mathbf{B}, \quad \mathbf{u} \rightleftarrows \boldsymbol{\varphi}, \quad k \in \mathbb{N}_0.
\end{aligned} \tag{5.16}$$

Here the notation $\mathbf{P} \to \boldsymbol{\mu}$, $\mathbf{C} \to \mathbf{D}$, $\mathbf{A} \to \mathbf{B}$, $\mathbf{u} \rightleftarrows \boldsymbol{\varphi}$ means that the next relation (5.16) is obtained from that already written out if the letters $\mathbf{C}$ and $\mathbf{A}$ are replaced by $\mathbf{D}$ and $\mathbf{B}$, respectively, $\mathbf{u}$ is replaced by $\boldsymbol{\varphi}$, and conversely, $\boldsymbol{\varphi}$ is replaced by $\mathbf{u}$. In what follows, this notation is also used.

We note that $\underset{\approx}{\mathbf{C}}^{3^-}_{(1,N)} = 2(k+1)\underset{\approx}{\mathbf{C}} \cdot \big[\mathbf{r}^{3^-} - g^{3^-}_{P^+}\big(g^{P^-}_{M^-} + A^{P^-}_{M^+}\big)\mathbf{r}^{M^-}\big]$, $k \in \mathbb{N}_0$, and similar relations for $\underset{\approx}{\mathbf{A}}^{3^-}_{(1,N)}$, $\underset{\approx}{\mathbf{B}}^{3^-}_{(1,N)}$, and $\underset{\approx}{\mathbf{D}}^{3^-}_{(1,N)}$ can be obtained from this formula by replacing the letter $\mathbf{C}$ by $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{D}$, respectively.

Now we can easily obtain the systems of Fourier thermal conduction laws in moments. Indeed, by analogy with (5.10), from (3.35) we obtain the system of Fourier thermal conduction laws in moments of the zeroth approximation:

$$\overset{(k)}{\mathbf{q}}_{(0)} = \overset{(k)}{\mathbf{q}}_{(0,N)} + \boldsymbol{\Lambda}^{3^-}_{(0,k)} \overset{(+)}{T}{}' + \boldsymbol{\Lambda}^{3^-}_{(k)} \overset{(-)}{T}{}', \tag{5.17}$$

$$\begin{aligned}
&\overset{(k)}{\mathbf{q}}_{(0,N)} = -\boldsymbol{\Lambda}^{M^-} \Big\{ \partial_M \overset{(k)}{T} - g^{3^-}_{M^+} \Big[k\overset{(k)}{T} + 2(k+1) \Big(\sum_{p=k}^{N} \overset{(p)}{T} - \overset{(k)}{T} \Big) \Big] \Big\} - 2(k+1)\boldsymbol{\Lambda}^{3^-} \sum_{p=k}^{N} [1 - (-1)^{k+p}] \overset{(p)}{T},\\
&\boldsymbol{\Lambda}^{3^-}_{(0,k)} = -2(k+1)\underset{\sim}{\boldsymbol{\Lambda}} \cdot \big(\mathbf{r}^{3^-} - g^{3^-}_{M^+}\mathbf{r}^{M^-}\big), \quad \boldsymbol{\Lambda}^{3^-}_{(k)} = 2(k+1)(-1)^k \underset{\sim}{\boldsymbol{\Lambda}} \cdot \mathbf{r}^{3^-},\\
&\overset{(+)}{T}{}'(x') = \sum_{p=N+1}^{\infty} \overset{(p)}{T}(x'), \quad \overset{(-)}{T}{}'(x') = \sum_{p=N+1}^{\infty} (-1)^p \overset{(p)}{T}(x').
\end{aligned} \tag{5.18}$$

Further, from Fourier's thermal conduction law in the first approximation, which follows from (3.34) for $s = 1$, by analogy with (5.15), we obtain the following system of Fourier thermal conduction laws in moments of the first approximation:

$$\overset{(k)}{\mathbf{q}}_{(1)} = \overset{(k)}{\mathbf{q}}_{(1,N)} + \boldsymbol{\Lambda}^{3^-}_{(1,k)} \cdot \overset{(+)}{T}{}' + \boldsymbol{\Lambda}^{3^-}_{(k)} \cdot \overset{(-)}{T}{}', \tag{5.19}$$

$$\begin{aligned}\overset{(k)}{\mathbf{q}}_{(1,N)} = -\mathbf{\Lambda}^{M^-}\Big\{&\partial_M \overset{(k)}{T} + \frac{1}{4}A^{P^-}_{M^+}\partial_P\big(\overset{(k-1)}{T} + 2\overset{(k)}{T} + \overset{(k+1)}{T}\big) - g^{3^-}_{P^+}\Big[g^{P^-}_{M^-}\Big(k\overset{(k)}{T} + 2(k+1)\Big(\sum_{p=k}^{N}\overset{(p)}{T} - \overset{(k)}{T}\Big)\Big)\\ &+ \frac{1}{4}A^{P^-}_{M^+}\Big((k-1)\overset{(k-1)}{T} - 4(k+2)\overset{(k)}{T} - (k+3)\overset{(k+1)}{T} + 8(k+1)\sum_{p=k}^{N}\overset{(p)}{T}\Big)\Big]\Big\}\\ &- 2(k+1)\mathbf{\Lambda}^{3^-}\sum_{p=k}^{N}[1-(-1)^{k+p}]\overset{(p)}{T},\end{aligned} \tag{5.20}$$

$$\mathbf{\Lambda}^{3^-}_{(1,k)} = -2(k+1)\underset{\sim}{\mathbf{\Lambda}} \cdot \big[\mathbf{r}^{3^-} - g^{3^-}_{P^+}\big(g^{P^-}_{M^-} + A^{P^-}_{M^+}\big)\mathbf{r}^{M^-}\big], \quad k \in \mathbb{N}_0.$$

6. ON BOUNDARY AND INITIAL CONDITIONS IN THE MOMENT MTDS

We write out the boundary conditions of kinematic, physical, and heat content on the face surfaces and the lateral side for the new parametrization of the domain occupied by a thin solid. From the boundary conditions on the lateral side, we obtain the corresponding boundary conditions in moments. Moreover, we obtain the equations for determining the vector functions $\overset{(+)}{\mathbf{u}}{}'$, $\overset{(-)}{\mathbf{u}}{}'$, $\overset{(+)}{\boldsymbol{\varphi}}{}'$, $\overset{(-)}{\boldsymbol{\varphi}}{}'$, and the functions $\overset{(+)}{T}{}'$, $\overset{(-)}{T}{}'$ by using the given values on the face surfaces.

6.1. Boundary conditions on the face surfaces. Definition of the vector functions $\overset{(+)}{\mathbf{u}}{}'$, $\overset{(-)}{\mathbf{u}}{}'$, $\overset{(+)}{\boldsymbol{\varphi}}{}'$, $\overset{(-)}{\boldsymbol{\varphi}}{}'$, and the functions $\overset{(+)}{T}{}'$, $\overset{(-)}{T}{}'$

First, we consider the boundary conditions of physical content on the face surfaces and represent them for the new parametrization of the domain occupied by the thin solid.

Suppose that $\overset{(+)}{\mathbf{P}}$ and $\overset{(-)}{\mathbf{P}}$ are given stress vectors and $\overset{(+)}{\boldsymbol{\mu}}$ and $\overset{(-)}{\boldsymbol{\mu}}$ are given couple stress vectors on the face surfaces $\overset{(+)}{S}$ and $\overset{(-)}{S}$, respectively. Let $\overset{(+)}{\mathbf{n}}$ and $\overset{(-)}{\mathbf{n}}$ be the outward unit normals on $\overset{(+)}{S}$ and $\overset{(-)}{S}$, respectively. It is easy to see that $\overset{(-)}{\mathbf{n}}$ and $\overset{(+)}{\mathbf{n}}$ have the following form for the new parametrization:

$$\begin{aligned}&\overset{(-)}{\mathbf{n}} = -\frac{\mathbf{r}^{3^-}}{\sqrt{g^{3^-3^-}}}, \quad \overset{(+)}{\mathbf{n}} = \frac{\mathbf{r}^{3^+}}{\sqrt{g^{3^+3^+}}} = \frac{1}{\sqrt{g^{3^+3^+}}}\big(\mathbf{r}^{3^-} - g^{3^-}_{P^+}g^{P^+}_{M^-}\mathbf{r}^{M^-}\big),\\ &g^{P^+}_{M^-} = g^{P}_{M^-}\Big|_{x^3=1} = \overset{(\mp)}{\vartheta}{}^{-1}A^{P^+}_{M^-}, \quad \overset{(\mp)}{\vartheta} = \sqrt{\overset{(+)}{g}\overset{(-)}{g}{}^{-1}} = \overset{(\pm)}{\vartheta}{}^{-1} = \det\big(g^{J^-}_{I^+}\big),\\ &A^{P^+}_{M^-} = A^{P}_{M^-}\Big|_{x^3=1} = \epsilon^{PL}\epsilon_{MN}g^{N^-}_{L^+}, \quad g^{3^+}_{M^-} = g^{3}_{M^-}\Big|_{x^3=1} = -\overset{(\mp)}{\vartheta}{}^{-1}g^{3^-}_{P^+}A^{P^+}_{M^-},\\ &g^{3^+3^+} = g^{33}\Big|_{x^3=1} = g^{3^+}_{m^-}g^{3^+}_{n^-}g^{m^-n^-} = g^{3^-3^-} + g^{3^+}_{M^-}g^{3^+}_{N^-}g^{M^-N^-}.\end{aligned} \tag{6.1}$$

Then, using (6.1), we can represent the boundary conditions of physical content on the face surfaces of the thin solid in the form

$$\begin{aligned}&\mathbf{r}^{3^-}\cdot \overset{(-)}{\underset{\sim}{\mathbf{P}}} = -\sqrt{g^{3^-3^-}}\overset{(-)}{\mathbf{P}}, \quad \big(\mathbf{r}^{3^-} - g^{3^-}_{P^+}g^{P^+}_{M^-}\mathbf{r}^{M^-}\big)\cdot \overset{(+)}{\underset{\sim}{\mathbf{P}}} = \sqrt{g^{3^+3^+}}\overset{(+)}{\mathbf{P}},\\ &\overset{(-)}{\underset{\sim}{\mathbf{P}}} = \underset{\sim}{\mathbf{P}}\Big|_{x^3=0}, \quad \overset{(+)}{\underset{\sim}{\mathbf{P}}} = \underset{\sim}{\mathbf{P}}\Big|_{x^3=1}; \quad \underset{\sim}{\mathbf{P}} \to \boldsymbol{\mu}, \quad x' \in \overset{(-)}{S}.\end{aligned} \tag{6.2}$$

For nonisothermal processes on the face surfaces $\overset{(+)}{S}$ and $\overset{(-)}{S}$, the respective normal components $\overset{(+)}{q}$ and $\overset{(-)}{q}$ of the heat flux vector $\mathbf{q}$ can be given. Then, by analogy with (6.2), the boundary conditions (conditions of the second kind or Neumann type conditions) [25] on the face surfaces have the form

$$\begin{aligned}&\mathbf{r}^{3^-}\cdot \overset{(-)}{\mathbf{q}} = -\sqrt{g^{3^-3^-}}\overset{(-)}{q}, \quad \big(\mathbf{r}^{3^-} - g^{3^-}_{P^+}g^{P^+}_{M^-}\mathbf{r}^{M^-}\big)\cdot \overset{(+)}{\mathbf{q}} = \sqrt{g^{3^+3^+}}\overset{(+)}{q}, \quad x' \in \overset{(-)}{S},\\ &\overset{(-)}{\mathbf{q}} = \mathbf{q}\Big|_{x^3=0}, \quad \overset{(+)}{\mathbf{q}} = \mathbf{q}\Big|_{x^3=1}.\end{aligned} \tag{6.3}$$

The boundary conditions corresponding to heat exchange with the environment according to the Newton law (boundary conditions of third kind) [25] can also be given. In this case, the boundary conditions on $\overset{(+)}{S}$ and $\overset{(-)}{S}$, respectively, have the form

$$\mathbf{r}^{3^-}\cdot\overset{(-)}{\mathbf{q}} = -\sqrt{g^{3^-3^-}}\,\overset{(-)}{\beta}\big(\overset{(-)}{T}_c-\overset{(-)}{T}\big), \quad \big(\mathbf{r}^{3^-}-g^{3^-}_{P^+}g^{P^+}_{M^-}\mathbf{r}^{M^-}\big)\cdot\overset{(+)}{\mathbf{q}} = \sqrt{g^{3^+3^+}}\,\overset{(+)}{\beta}\big(\overset{(+)}{T}_c-\overset{(+)}{T}\big), \quad x'\in\overset{(-)}{S}, \tag{6.4}$$

where T_c is the given temperature of the environment, β is the convective heat transfer coefficient (cal/cm^2·s·grad), and

$$\overset{(-)}{\beta}=\beta\Big|_{x^3=0}, \quad \overset{(+)}{\beta}=\beta\Big|_{x^3=1}, \quad \overset{(-)}{T}=T\Big|_{x^3=0}, \quad \overset{(+)}{T}=T\Big|_{x^3=1}, \quad \overset{(-)}{T}_c=T_c\Big|_{x^3=0}, \quad \overset{(+)}{T}_c=T_c\Big|_{x^3=1}. \tag{6.5}$$

In what follows, we consider the boundary conditions of first kind and the Dirichlet type conditions.

Now we obtain equations for the vector functions $\overset{(+)}{\mathbf{u}}{}'$, $\overset{(-)}{\mathbf{u}}{}'$ and $\overset{(+)}{\boldsymbol{\varphi}}{}'$, $\overset{(-)}{\boldsymbol{\varphi}}{}'$ as well as the functions $\overset{(+)}{T}$ and $\overset{(-)}{T}$. In this connection, we note that we seek an approximate solution of an approximate problem, which contains the system of equations in moments of the (r, N)th approximation with boundary conditions on the lateral face (the statements of the problems and the initial conditions will be considered later). Moreover, it follows from (5.12), (5.15) and (5.17), (5.19) that the forms of representation of systems of Hooke's laws and Fourier's thermal conduction laws are the same. (They remain to be the same for approximations of an arbitrary order.) The difference is only in notation. In this case, the methods for determining the desired functions are the same and are similar to the methods for determining similar functions studied in [23]. Therefore, in what follows we consider a method for deriving the system of equations for these functions in the case of systems of Hooke's laws (5.12) and Fourier's thermal conduction laws (5.17) in moments of the zeroth approximation.

We assume that the stress and couple stress tensors have the following representations:

$$\underset{\sim}{\mathbf{P}}_{(0)}(x',x^3)=\sum_{k=0}^{N}\overset{(k)}{\underset{\sim}{\mathbf{P}}}_{(0)}(x')\hat{U}^*_k(x^3), \quad \underset{\sim}{\boldsymbol{\mu}}_{(0)}(x',x^3)=\sum_{k=0}^{N}\overset{(k)}{\underset{\sim}{\boldsymbol{\mu}}}_{(0)}(x')\hat{U}^*_k(x^3). \tag{6.6}$$

Then, owing to (6.6), the boundary conditions of physical content on the face surfaces (6.2) can be written in the form

$$\mathbf{r}^{3^-}\cdot\overset{(-)}{\underset{\sim}{\mathbf{P}}}_{(0)} = -\sqrt{g^{3^-3^-}}\,\overset{(-)}{\mathbf{P}}, \quad \big(\mathbf{r}^{3^-}-g^{3^-}_{P^+}g^{P^+}_{M^-}\mathbf{r}^{M^-}\big)\cdot\overset{(+)}{\underset{\sim}{\mathbf{P}}}_{(0)} = \sqrt{g^{3^+3^+}}\,\overset{(+)}{\mathbf{P}}, \quad \mathbf{P}\to\boldsymbol{\mu}, \quad x'\in\overset{(-)}{S}. \tag{6.7}$$

Taking into account the values of the orthonormal shifted Chebyshev polynomials of the second kind at the endpoints of the interval $[0,1]$,

$$\hat{U}^*_k(0)=(-1)^k\frac{2}{\sqrt{\pi}}(k+1), \quad \hat{U}^*_k(1)=\frac{2}{\sqrt{\pi}}(k+1), \tag{6.8}$$

from (6.6) we obtain

$$\overset{(-)}{\underset{\sim}{\mathbf{P}}}_{(0)}=\underset{\sim}{\mathbf{P}}_{(0)}\Big|_{x^3=0}=\frac{2}{\sqrt{\pi}}\sum_{k=0}^{N}(-1)^k(k+1)\overset{(k)}{\underset{\sim}{\mathbf{P}}}_{(0)}, \quad \overset{(+)}{\underset{\sim}{\mathbf{P}}}_{(0)}=\underset{\sim}{\mathbf{P}}_{(0)}\Big|_{x^3=1}=\frac{2}{\sqrt{\pi}}\sum_{k=0}^{N}(k+1)\overset{(k)}{\underset{\sim}{\mathbf{P}}}_{(0)}, \quad \mathbf{P}\to\boldsymbol{\mu}.$$

Using (5.12), (5.14), and the last formulas, we find

$$\begin{gathered}
b_{(N)}\underset{\approx}{\mathbf{C}}\cdot\cdot\big(\mathbf{r}^{3^-}-g^{3^-}_{M^+}\mathbf{r}^{M^-}\big)\overset{(+)}{\mathbf{u}}{}'-a_{(N)}\underset{\approx}{\mathbf{C}}\cdot\cdot\mathbf{r}^{3^-}\overset{(-)}{\mathbf{u}}{}'+b_{(N)}\underset{\approx}{\mathbf{A}}\cdot\cdot\big(\mathbf{r}^{3^-}-g^{3^-}_{M^+}\mathbf{r}^{M^-}\big)\overset{(+)}{\boldsymbol{\varphi}}{}'-a_{(N)}\underset{\approx}{\mathbf{A}}\cdot\cdot\mathbf{r}^{3^-}\overset{(-)}{\boldsymbol{\varphi}}{}'\\
=\frac{\sqrt{\pi}}{2}\overset{(-)}{\underset{\sim}{\mathbf{P}}}_{(0)}-\sum_{k=0}^{N}(-1)^k(k+1)\overset{(k)}{\underset{\sim}{\mathbf{P}}}_{(0,N)},\\
a_{(N)}\underset{\approx}{\mathbf{C}}\cdot\cdot\big(\mathbf{r}^{3^-}-g^{3^-}_{M^+}\mathbf{r}^{M^-}\big)\overset{(+)}{\boldsymbol{\varphi}}{}'-b_{(N)}\underset{\approx}{\mathbf{C}}\cdot\cdot\mathbf{r}^{3^-}\overset{(-)}{\mathbf{u}}{}'+a_{(N)}\underset{\approx}{\mathbf{A}}\cdot\cdot\big(\mathbf{r}^{3^-}-g^{3^-}_{M^+}\mathbf{r}^{M^-}\big)\overset{(+)}{\boldsymbol{\varphi}}{}'-b_{(N)}\underset{\approx}{\mathbf{A}}\cdot\cdot\mathbf{r}^{3^-}\overset{(-)}{\mathbf{u}}{}'\\
=\frac{\sqrt{\pi}}{2}\overset{(+)}{\underset{\sim}{\mathbf{P}}}_{(0)}-\sum_{k=0}^{N}(k+1)\overset{(k)}{\underset{\sim}{\mathbf{P}}}_{(0,N)},\\
\mathbf{C}\to\mathbf{D}, \quad \mathbf{A}\to\mathbf{B}, \quad \mathbf{D}\to\boldsymbol{\mu}, \quad \mathbf{u}\rightleftarrows\boldsymbol{\varphi},
\end{gathered} \tag{6.9}$$

$$a_{(N)}=2\sum_{k=0}^{N}(k+1)^2=\frac{1}{3}(N+1)(N+2)(2N+3),\quad b_{(N)}=2\sum_{k=0}^{N}(-1)^k(k+1)^2. \tag{6.10}$$

By multiplying the first and third (unwritten) relations in (6.9) in the sense of the inner product on the left by $\mathbf{r}^{3^-}$, by multiplying the second and fourth (unwritten) relations by $\big(\mathbf{r}^{3^-}-g^{3^-}_{P^+}g^{P^+}_{K^-}\mathbf{r}^{K^-}\big)$, and by taking into account (6.7), we obtain the desired equations

$$\begin{gathered}
\overset{(+)}{\underset{\sim}{\mathbf{C}}}{}'_{(0,N)}\cdot\overset{(+)}{\mathbf{u}}{}'+\overset{(-)}{\underset{\sim}{\mathbf{C}}}{}'_{(0,N)}\cdot\overset{(-)}{\mathbf{u}}{}'+\overset{(+)}{\underset{\sim}{\mathbf{A}}}{}'_{(0,N)}\cdot\overset{(+)}{\boldsymbol{\varphi}}{}'+\overset{(-)}{\underset{\sim}{\mathbf{A}}}{}'_{(0,N)}\cdot\overset{(-)}{\boldsymbol{\varphi}}{}'=\overset{(-)}{\mathbf{A}}_{(0,N)},\\
\overset{(+)}{\underset{\sim}{\mathbf{C}}}{}''_{(0,N)}\cdot\overset{(+)}{\mathbf{u}}{}'+\overset{(-)}{\underset{\sim}{\mathbf{C}}}{}''_{(0,N)}\cdot\overset{(-)}{\mathbf{u}}{}'+\overset{(+)}{\underset{\sim}{\mathbf{A}}}{}''_{(0,N)}\cdot\overset{(+)}{\boldsymbol{\varphi}}{}'+\overset{(-)}{\underset{\sim}{\mathbf{A}}}{}''_{(0,N)}\cdot\overset{(-)}{\boldsymbol{\varphi}}{}'=\overset{(+)}{\mathbf{A}}_{(0,N)},\\
\overset{(+)}{\underset{\sim}{\mathbf{B}}}{}'_{(0,N)}\cdot\overset{(+)}{\mathbf{u}}{}'+\overset{(-)}{\underset{\sim}{\mathbf{B}}}{}'_{(0,N)}\cdot\overset{(-)}{\mathbf{u}}{}'+\overset{(+)}{\underset{\sim}{\mathbf{D}}}{}'_{(0,N)}\cdot\overset{(+)}{\boldsymbol{\varphi}}{}'+\overset{(-)}{\underset{\sim}{\mathbf{D}}}{}'_{(0,N)}\cdot\overset{(-)}{\boldsymbol{\varphi}}{}'=\overset{(-)}{\mathbf{B}}_{(0,N)},\\
\overset{(+)}{\underset{\sim}{\mathbf{B}}}{}''_{(0,N)}\cdot\overset{(+)}{\mathbf{u}}{}'+\overset{(-)}{\underset{\sim}{\mathbf{B}}}{}''_{(0,N)}\cdot\overset{(-)}{\mathbf{u}}{}'+\overset{(+)}{\underset{\sim}{\mathbf{D}}}{}''_{(0,N)}\cdot\overset{(+)}{\boldsymbol{\varphi}}{}'+\overset{(-)}{\underset{\sim}{\mathbf{D}}}{}''_{(0,N)}\cdot\overset{(-)}{\boldsymbol{\varphi}}{}'=\overset{(+)}{\mathbf{B}}_{(0,N)},\\
\overset{(+)}{\underset{\sim}{\mathbf{C}}}{}'_{(0,N)}=b_{(N)}\big(\underset{\sim}{\mathbf{C}}^{3^-\cdot\cdot 3^-}-g^{3^-}_{M^+}\underset{\sim}{\mathbf{C}}^{3^-\cdot\cdot M^-}\big),\quad \overset{(-)}{\underset{\sim}{\mathbf{C}}}{}'_{(0,N)}=-a_{(N)}\underset{\sim}{\mathbf{C}}^{3^-\cdot\cdot 3^-},\\
\overset{(+)}{\underset{\sim}{\mathbf{C}}}{}''_{(0,N)}=a_{(N)}\Big[\big(\underset{\sim}{\mathbf{C}}^{3^-\cdot\cdot 3^-}-g^{3^-}_{M^+}\underset{\sim}{\mathbf{C}}^{3^-\cdot\cdot M^-}\big)-g^{3^-}_{P^+}g^{P^+}_{K^-}\big(\underset{\sim}{\mathbf{C}}^{K^-\cdot\cdot 3^-}-g^{3^-}_{M^+}\underset{\sim}{\mathbf{C}}^{K^-\cdot\cdot M^-}\big)\Big],\\
\overset{(-)}{\underset{\sim}{\mathbf{C}}}{}''_{(0,N)}=-b_{(N)}\big(\underset{\sim}{\mathbf{C}}^{3^-\cdot\cdot 3^-}-g^{3^-}_{P^+}g^{P^+}_{K^-}\underset{\sim}{\mathbf{C}}^{K^-\cdot\cdot 3^-}\big),\quad \overset{(k)}{\mathbf{P}}{}^{m^-\cdot}_{(0,N)}=\mathbf{r}^{m^-}\cdot\overset{(k)}{\underset{\sim}{\mathbf{P}}}_{(0,N)},\quad \overset{(k)}{\boldsymbol{\mu}}{}^{m^-\cdot}_{(0,N)}=\mathbf{r}^{m^-}\cdot\overset{(k)}{\underset{\sim}{\boldsymbol{\mu}}}_{(0,N)},\\
\overset{(-)}{\mathbf{A}}_{(0,N)}=-\Big[\frac{\sqrt{\pi}}{2}\sqrt{g^{3^-3^-}}\overset{(-)}{\mathbf{P}}+\sum_{k=0}^{N}(-1)^k(k+1)\overset{(k)}{\mathbf{P}}{}^{3^-\cdot}_{(0,N)}\Big],\quad \underset{\sim}{\mathbf{A}}^{m^-\cdot\cdot n^-}=\mathbf{r}^{m^-}\cdot\underset{\approx}{\mathbf{A}}\cdot\mathbf{r}^{n^-},\\
\overset{(+)}{\mathbf{A}}_{(0,N)}=\frac{\sqrt{\pi}}{2}\sqrt{g^{3^+3^+}}\overset{(+)}{\mathbf{P}}-\sum_{k=0}^{N}(k+1)\big(\overset{(k)}{\mathbf{P}}{}^{3^-\cdot}_{(0,N)}-g^{3^-}_{P^+}g^{P^+}_{K^-}\overset{(k)}{\mathbf{P}}{}^{K^-\cdot}_{(0,N)}\big),\quad \underset{\sim}{\mathbf{B}}^{m^-\cdot\cdot n^-}=\mathbf{r}^{m^-}\cdot\underset{\approx}{\mathbf{B}}\cdot\mathbf{r}^{n^-},\\
\overset{(-)}{\mathbf{B}}_{(0,N)}=-\Big[\frac{\sqrt{\pi}}{2}\sqrt{g^{3^-3^-}}\overset{(-)}{\boldsymbol{\mu}}+\sum_{k=0}^{N}(-1)^k(k+1)\overset{(k)}{\boldsymbol{\mu}}{}^{3^-\cdot}_{(0,N)}\Big],\quad \underset{\sim}{\mathbf{C}}^{m^-\cdot\cdot n^-}=\mathbf{r}^{m^-}\cdot\underset{\approx}{\mathbf{C}}\cdot\mathbf{r}^{n^-},\\
\overset{(+)}{\mathbf{B}}_{(0,N)}=\frac{\sqrt{\pi}}{2}\sqrt{g^{3^+3^+}}\overset{(+)}{\boldsymbol{\mu}}-\sum_{k=0}^{N}(k+1)\big(\overset{(k)}{\boldsymbol{\mu}}{}^{3^-\cdot}_{(0,N)}-g^{3^-}_{P^+}g^{P^+}_{K^-}\overset{(k)}{\boldsymbol{\mu}}{}^{K^-\cdot}_{(0,N)}\big),\quad \underset{\sim}{\mathbf{D}}^{m^-\cdot\cdot n^-}=\mathbf{r}^{m^-}\cdot\underset{\approx}{\mathbf{D}}\cdot\mathbf{r}^{n^-},
\end{gathered}\tag{6.11}$$

The expressions for $\overset{(+)}{\underset{\sim}{\mathbf{A}}}{}'_{(0,N)}$, $\overset{(-)}{\underset{\sim}{\mathbf{A}}}{}'_{(0,N)}$, $\overset{(+)}{\underset{\sim}{\mathbf{A}}}{}''_{(0,N)}$, $\overset{(-)}{\underset{\sim}{\mathbf{A}}}{}''_{(0,N)}$ ($\overset{(+)}{\underset{\sim}{\mathbf{B}}}{}'_{(0,N)}$, $\overset{(-)}{\underset{\sim}{\mathbf{B}}}{}'_{(0,N)}$, $\overset{(+)}{\underset{\sim}{\mathbf{B}}}{}''_{(0,N)}$, $\overset{(-)}{\underset{\sim}{\mathbf{B}}}{}''_{(0,N)}$) and $\overset{(+)}{\underset{\sim}{\mathbf{D}}}{}'_{(0,N)}$, $\overset{(-)}{\underset{\sim}{\mathbf{D}}}{}'_{(0,N)}$, $\overset{(+)}{\underset{\sim}{\mathbf{D}}}{}''_{(0,N)}$, $\overset{(-)}{\underset{\sim}{\mathbf{D}}}{}''_{(0,N)}$ are obtained from the expressions for $\overset{(+)}{\underset{\sim}{\mathbf{C}}}{}'_{(0,N)}$, $\overset{(-)}{\underset{\sim}{\mathbf{C}}}{}'_{(0,N)}$, $\overset{(+)}{\underset{\sim}{\mathbf{C}}}{}''_{(0,N)}$, and $\overset{(-)}{\underset{\sim}{\mathbf{C}}}{}''_{(0,N)}$ by replacing the letter $\mathbf{C}$ by the letters $\mathbf{A}$ ($\mathbf{B}$) and $\mathbf{D}$, respectively.

Relations (6.11) form an algebraic system of twelve equations for the twelve unknowns $\overset{(+)}{\mathbf{u}}{}'$, $\overset{(-)}{\mathbf{u}}{}'$, $\overset{(+)}{\boldsymbol{\varphi}}{}'$, and $\overset{(-)}{\boldsymbol{\varphi}}{}'$ (four vectors). By solving this system, we obtain the vectors $\overset{(+)}{\mathbf{u}}{}'$, $\overset{(-)}{\mathbf{u}}{}'$, $\overset{(+)}{\boldsymbol{\varphi}}{}'$, and $\overset{(-)}{\boldsymbol{\varphi}}{}'$ expressed in terms of the moments $\overset{(m)}{\mathbf{u}}$, $\partial_I\overset{(m)}{\mathbf{u}}$, $\overset{(m)}{\boldsymbol{\varphi}}$ and $\partial_I\overset{(m)}{\boldsymbol{\varphi}}$, $\overset{(m)}{\vartheta}$; $m=\overline{0,\,N}$. If we take account of the obtained expressions for the desired vectors in (5.12), we find the CE (the systems of Hooke's laws) in moments of the zeroth approximation. Moreover, $\overset{(m)}{\underset{\sim}{\mathbf{P}}}_{(0)}$ and $\overset{(k)}{\underset{\sim}{\boldsymbol{\mu}}}_{(0)}$ are linear forms in $\overset{(m)}{\mathbf{u}}$, $\partial_I\overset{(m)}{\mathbf{u}}$, $\overset{(m)}{\boldsymbol{\varphi}}$ and $\partial_I\overset{(m)}{\boldsymbol{\varphi}}$, $\overset{(m)}{\vartheta}$; $m=\overline{0,\,N}$. Substituting $\overset{(k)}{\underset{\sim}{\mathbf{P}}}_{(0)}$ and $\overset{(k)}{\underset{\sim}{\boldsymbol{\mu}}}_{(0)}$ into (6.6), we obtain approximate expressions for the stress and couple stress tensors which satisfy the boundary conditions on the face surfaces for any vector fields $\overset{(m)}{\mathbf{u}}$, $\overset{(m)}{\boldsymbol{\varphi}}$ and scalar fields $\overset{(m)}{\vartheta}$; $m=\overline{0,\,N}$, which are moments of the desired vector fields $\mathbf{u}$, $\boldsymbol{\varphi}$, and the scalar field ϑ. Following Vekua, we call the expressions for $\overset{(k)}{\underset{\sim}{\mathbf{P}}}_{(0)}$ and $\overset{(k)}{\underset{\sim}{\boldsymbol{\mu}}}_{(0)}$, consistent with the boundary conditions on the face surfaces, the normalized kth moments of the stress and couple stress tensors in the zeroth approximation. (The normalized kth moments of the stress and couple stress tensor fields can be found in a similar way for an arbitrary approximation.)

Now we obtain equations for the functions $\overset{(+)}{T}{}'$ and $\overset{(-)}{T}{}'$. We assume that the heat influx vector is given by the approximate formula

$$\mathbf{q}_{(0)}(x',x^3)=\sum_{r=0}^{N}\overset{(k)}{\mathbf{q}}_{(0)}(x')\hat{U}_0^*(x^3). \tag{6.12}$$

First, consider the boundary conditions of the second kind (the Neumann type conditions) (6.3). By analogy with (6.7), we represent them in the form

$$\mathbf{r}^{3^-}\cdot\overset{(-)}{\mathbf{q}}_{(0)}=\sqrt{g^{3^-3^-}}\,\overset{(-)}{q},\quad \big(\mathbf{r}^{3^-}-g^{3^-}_{P^+}g^{P^+}_{M^-}\mathbf{r}^{M^-}\big)\cdot\overset{(+)}{\mathbf{q}}_{(0)}=\sqrt{g^{3^+3^+}}\,\overset{(+)}{q},\quad x'\in\overset{(-)}{S}. \tag{6.13}$$

By (6.8), from (6.12) we obtain

$$\overset{(-)}{\mathbf{q}}_{(0)}=\underset{\sim}{\mathbf{q}}_{(0)}\Big|_{x^3=0}=\frac{2}{\sqrt{\pi}}\sum_{k=0}^{N}(-1)^k(k+1)\overset{(k)}{\mathbf{q}}_{(0)},\quad \overset{(+)}{\mathbf{q}}_{(0)}=\underset{\sim}{\mathbf{q}}_{(0)}\Big|_{x^3=1}=\frac{2}{\sqrt{\pi}}\sum_{k=0}^{N}(k+1)\overset{(k)}{\mathbf{q}}_{(0)}. \tag{6.14}$$

Taking into account (5.17) and the second and third relations in (5.18) and (6.10), from (6.14) we obtain

$$\begin{aligned}
&-b_{(N)}\underset{\sim}{\mathbf{\Lambda}}\cdot\big(\mathbf{r}^{3^-}-g^{3^-}_{M^+}\mathbf{r}^{M^-}\big)\overset{(+)}{T}{}'+a_{(N)}\underset{\sim}{\mathbf{\Lambda}}\cdot\mathbf{r}^{3^-}\overset{(-)}{T}{}'=\frac{\sqrt{\pi}}{2}\overset{(-)}{\mathbf{q}}_{(0)}-\sum_{k=0}^{N}(-1)^k(k+1)\overset{(k)}{\mathbf{q}}_{(0,N)},\\
&-a_{(N)}\underset{\sim}{\mathbf{\Lambda}}\cdot\big(\mathbf{r}^{3^-}-g^{3^-}_{M^+}\mathbf{r}^{M^-}\big)\overset{(+)}{T}{}'+b_{(N)}\underset{\sim}{\mathbf{\Lambda}}\cdot\mathbf{r}^{3^-}\overset{(-)}{T}{}'=\frac{\sqrt{\pi}}{2}\overset{(+)}{\mathbf{q}}_{(0)}-\sum_{k=0}^{N}(k+1)\overset{(k)}{\mathbf{q}}_{(0,N)}.
\end{aligned} \tag{6.15}$$

By multiplying the first relation in (6.15) in the sense of the inner product by $\mathbf{r}^{3^-}$ and the second relation by $\mathbf{r}^{3^-}-g^{3^-}_{P^+}g^{P^+}_{K^-}\mathbf{r}^{K^-}$ and by using (6.13), we obtain the desired system of equations

$$\overset{(+)}{\Lambda}{}'_{(0,N)}\overset{(+)}{T}{}'+\overset{(-)}{\Lambda}{}'_{(0,N)}\overset{(-)}{T}{}'=\overset{(-)}{Q}_{(0,N)},\quad \overset{(+)}{\Lambda}{}''_{(0,N)}\overset{(+)}{T}{}'+\overset{(-)}{\Lambda}{}''_{(0,N)}\overset{(-)}{T}{}'=\overset{(+)}{Q}_{(0,N)}, \tag{6.16}$$

where

$$\begin{aligned}
&\overset{(+)}{\Lambda}{}'_{(0,N)}=-b_{(N)}\big(\Lambda^{3^-3^-}-g^{3^-}_{M^+}\Lambda^{3^-M^-}\big),\quad \overset{(-)}{\Lambda}{}'_{(0,N)}=a_{(N)}\Lambda^{3^-3^-},\\
&\Lambda^{m^-n^-}=\mathbf{r}^{m^-}\cdot\underset{\sim}{\mathbf{\Lambda}}\cdot\mathbf{r}^{n^-},\quad \overset{(k)}{q}{}^{m^-}_{(0,N)}=\overset{(k)}{\mathbf{q}}_{(0,N)}\cdot\mathbf{r}^{m^-},\\
&\overset{(+)}{\Lambda}{}''_{(0,N)}=-a_{(N)}\Big[\big(\Lambda^{3^-3^-}-g^{3^-}_{M^+}\Lambda^{3^-M^-}\big)-g^{3^-}_{P^+}g^{P^+}_{K^-}\big(\Lambda^{K^-3^-}-g^{3^-}_{M^+}\Lambda^{K^-M^-}\big)\Big],\\
&\overset{(-)}{\Lambda}{}''_{(0,N)}=b_{(N)}\big(\Lambda^{3^-3^-}-g^{3^-}_{P^+}g^{P^+}_{K^-}\Lambda^{K^-3^-}\big),\\
&\overset{(-)}{Q}_{(0,N)}=-\Big[\frac{\sqrt{\pi}}{2}\sqrt{g^{3^-3^-}}\,\overset{(-)}{q}+\sum_{k=0}^{N}(-1)^k(k+1)\overset{(k)}{q}{}^{3^-}_{(0,N)}\Big],\\
&\overset{(+)}{Q}_{(0,N)}=\frac{\sqrt{\pi}}{2}\sqrt{g^{3^+3^+}}\,\overset{(+)}{q}-\sum_{k=0}^{N}(k+1)\big(\overset{(k)}{q}{}^{3^-}_{(0,N)}-g^{3^-}_{P^+}g^{P^+}_{K^-}\overset{(k)}{q}{}^{K^-}_{(0,N)}\big).
\end{aligned}$$

Relations (6.16) form a system of equations for the two unknown variables $\overset{(+)}{T}{}'$ and $\overset{(-)}{T}{}'$. By solving this system, we obtain the functions $\overset{(+)}{T}{}'$ and $\overset{(-)}{T}{}'$ expressed via the moments $\overset{(m)}{T}$, $\partial_I\overset{(m)}{T}$; $m=\overline{0,\,N}$. Substituting the obtained expressions for $\overset{(+)}{T}{}'$ and $\overset{(-)}{T}{}'$ into (5.17), we find relations in moments between $\overset{(k)}{\mathbf{q}}_{(0)}$ and $\overset{(m)}{T}$, $\partial_I\overset{(m)}{T}$; $m=\overline{0,\,N}$. These relations are linear forms in $\overset{(m)}{T}$, $\partial_I\overset{(m)}{T}$; $m=\overline{0,\,N}$. Taking into account the expressions for $\overset{(k)}{\mathbf{q}}_{(0)}$ in (6.12), we obtain an approximate expression for the heat influx flow satisfying the boundary conditions of the second kind on the face surfaces for any scalar fields $\overset{(m)}{T}$, $m = \overline{0,\ N}$, which are moments of the desired scalar field $T(x', x^3)$. Following Vekua, we call the expression for $\overset{(k)}{\mathbf{q}}_{(0)}$ consistent with the boundary conditions on the face surfaces the normalized kth-order moment of the field of the heat influx vector in the zeroth approximation. (The normalized kth-order moment of the heat influx vector in an arbitrary approximation is determined in a similar way.)

Now we consider the boundary conditions of the third kind (conditions of heat exchange with the environment according to the Newton law). In this case, we need not derive the system of equations for the functions $\overset{(+)}{T}{}'$ and $\overset{(-)}{T}{}'$ in detail. Indeed, by representing the boundary conditions (6.4) in the form similar to (6.13),

$$\mathbf{r}^{3^-}\cdot\overset{(k)}{\mathbf{q}}_{(0)} = -\sqrt{g^{3^-3^-}}\,\overset{(-)}{\beta}\big(\overset{(-)}{T}_c - \overset{(-)}{T}\big), \quad \big(\mathbf{r}^{3^-} - g^{3^-}_{P^+} g^{P^+}_{M^-}\mathbf{r}^{M^-}\big)\cdot \overset{(k)}{\mathbf{q}}_{(0)} = \sqrt{g^{3^+3^+}}\,\overset{(+)}{\beta}\big(\overset{(+)}{T}_c - \overset{(+)}{T}\big), \quad x'\in\overset{(-)}{S},$$

we can easily see that for $\overset{(+)}{T}{}'$ and $\overset{(-)}{T}{}'$ we obtain equations similar to (6.16) whose right-hand sides are obtained from the right-hand sides of (6.16) by replacing $\overset{(-)}{q}$ and $\overset{(+)}{q}$ in them by $\overset{(-)}{\beta}\big(\overset{(-)}{T}_c - \overset{(-)}{T}\big)$ and $\overset{(+)}{\beta}\big(\overset{(+)}{T}_c - \overset{(+)}{T}\big)$, respectively. For brevity, here we do not write out these equations.

We note that, in the simplified scheme of reduction to the system of equations of a finite order, just as above, for the system of equations (of motion or heat influx) we take the system of equations (of motion or heat influx) in moments of the (r, N)th approximation, where r and N are some fixed nonnegative integers, and then, in Hooke's laws and Fourier's thermal conduction laws in moments of the rth approximation, we assume that

$$\overset{(k)}{\mathbf{u}} = 0,\ \overset{(k)}{\boldsymbol{\varphi}} = 0,\ \overset{(k)}{\vartheta} = 0, \text{ and } \overset{(k)}{T} = 0 \text{ if } k \geq N.$$

For example, in the cases $r = 0$ and $r = 1$, for the system of Hooke's law one must take relations (5.13) and (5.16) for $k = (\overline{0,\ N})$, and for the system of Fourier's thermal conduction laws one must take the first relations in (5.18) and (5.20) for $k = (\overline{0,\ N})$.

In what follows, we note that, in the simplified scheme of reduction, for each approximate solution of the boundary value problem, just as it was done in [23] for the classical version of the theory with the use of the Legendre polynomials, we construct a correction term which ensures that the boundary conditions are satisfied on the face surfaces. For brevity, in this paper we do not consider the construction of the correction term for various boundary conditions on the face surfaces.

It follows from the above that the three-dimensional Hooke's and Fourier's thermal conduction laws in the theory of thin solids are replaced by the corresponding infinite systems of laws in moments. In this case, each law contains infinitely many terms. Therefore, by analogy with systems of equations of motion and heat influx in moments, we should reduce them to finite systems of laws in moments such that each of these laws contains finitely many terms. This reduction is performed as follows: we fix some (in particular, the same numbers as in the reduction of systems of equations) nonnegative integers r and N and then, from the infinite system of laws in normalized moments of the stress and couple stress tensors in the rth approximation, we take the set of the first $N + 1$ laws. In the simplified scheme of reduction, from the infinite system of laws in moments of the rth approximation, we the take the set of the first $N + 1$ laws and, in each of these laws, we neglect the moments of the desired variables whose order exceeds N. In this connection, it is expedient to introduce the following definitions.

Definition 6.1. The set of Hooke's laws (Fourier's thermal conduction laws) in moments consisting of the first $N + 1$ laws of the corresponding infinite systems of Hooke's laws (Fourier's thermal conduction laws) in normalized moments of the stress and couple stress tensors of order r is called the system of Hooke's laws (Fourier's thermal conduction laws) in normalized moments of the stress and couple stress tensors of the (r, N)th approximation.

Definition 6.2. The set of Hooke's laws (Fourier's thermal conduction laws) in moments consisting of the first $N + 1$ laws of the corresponding infinite systems of Hooke's laws (Fourier's thermal conduction laws) in normalized moments of order r such that each law does not contain the desired variables whose order exceeds N is called the system of Hooke's laws (Fourier's thermal conduction laws) in moments of the (r, N)th approximation.

For example, by these definitions, the system of Hooke's laws (Fourier's thermal conduction laws) in normalized moments of the stress and couple stress (the vector of heat influx) tensors of the $(0, N)$th approximation is obtained from (5.12) (or (5.17)) by choosing the first $N+1$ relations and by taking into account the expressions for $\overset{(+)}{\mathbf{u}}$, $\overset{(-)}{\mathbf{u}}$, $\overset{(+)}{\boldsymbol{\varphi}}$, and $\overset{(-)}{\boldsymbol{\varphi}}$ ($\overset{(+)}{T}$ and $\overset{(-)}{T}$) obtained by solving the systems of equations (6.11) (or (6.16)).

For example, in a similar way, the system of Hooke's laws (Fourier's thermal conduction laws) in moments of the $(0, N)$th approximation is obtained from (5.13) (relations in the first row in (5.18)) by taking the first $N+1$ relations. Hence the choice of the first $N+1$ relations means that $k = \overline{0, N}$.

6.2. Boundary conditions in moments in the theory of thin solids

To obtain well-posed statements of problems in the theory of thin solids, any system of equations, consistent or inconsistent (in the simplified scheme of reduction to a finite-order system) with the boundary conditions on the face surfaces, should be supplemented with the boundary conditions on the contour $\partial\overset{(-)}{S}$ of the main base surface $\overset{(-)}{S}$.

On the lateral face Σ, kinematic conditions (the displacement and rotation vectors) or static conditions (the stress and couple stress vectors) can be given. The kinematic conditions can be given on one part, Σ_1, and the static conditions can be given on the other part, Σ_2 ($\Sigma_1 \cup \Sigma_2 = \Sigma$, $\Sigma_1 \cap \Sigma_2 = \varnothing$). In the case of nonisothermal processes, the boundary conditions of heat content of the first kind (of Dirichlet type) or of the second kind (of Neumann type) or even of third kind (the heat exchange with the environment according to the Newton law) can also be posed on part of the lateral face. In what follows, we consider the kinematic, physical, and thermal boundary conditions on the lateral face and use them to obtain the corresponding boundary conditions in moments on the boundary contour of the main base surface.

In what follows, we assume that the lateral face Σ consists of ruled surfaces and some nonnegative integers r and N are fixed. The fact that such numbers are given means that we consider systems of equations in moments of the (r, N)th approximation, systems of Hooke's laws and Fourier's thermal conduction laws in normalized moments or in moments of the (r, N)th approximation, as well as the corresponding boundary conditions on the boundary contour of the main base surface and the initial conditions in moments. Then, obviously, the unknowns are the moments $\overset{(m)}{\underset{\sim}{\mathbf{P}}}$, $\overset{(m)}{\underset{\sim}{\boldsymbol{\mu}}}$, $\overset{(m)}{\mathbf{u}}$, $\overset{(m)}{\boldsymbol{\varphi}}$, $\overset{(m)}{T}$; $m = \overline{0, N}$. For example, for the moment theory of thin solids in nonisothermal processes, the problem is well posed if $2N+2$ vectors of kinematic boundary conditions are posed on the boundary contour $\partial\overset{(-)}{S}$ of the main base surface $\overset{(-)}{S}$ and $N+1$ boundary conditions of heat content are posed on the part $\partial\overset{(-)}{S}_q \subseteq \partial\overset{(-)}{S}$ (in the case of the first boundary value problem); or $2N+2$ vector static boundary conditions are posed on $\partial\overset{(-)}{S}$ and $N+1$ boundary conditions of heat content are posed on the part $\partial\overset{(-)}{S}_q \subseteq \partial\overset{(-)}{S}$ (in the case of the second boundary value problem); or $2N+2$ vector kinematic boundary conditions are posed on the part $\partial\overset{(-)}{S}_1$ of the contour and $2N+2$ vector static boundary conditions are posed on the other part $\partial\overset{(-)}{S}_2$ ($\partial\overset{(-)}{S}_1 \cup \partial\overset{(-)}{S}_2 = \partial\overset{(-)}{S}$, $\partial\overset{(-)}{S}_1 \cap \partial\overset{(-)}{S}_2 = \varnothing$), while $N+1$ boundary conditions of heat content are posed on $\partial\overset{(-)}{S}_q \subseteq \partial\overset{(-)}{S}$ (in the case of the mixed boundary value problem). We note that, in the case of dynamic problems, the boundary conditions in moments must be supplemented with initial conditions in moments, which we consider later.

6.3. Kinematic boundary conditions in moments

Suppose that the displacement vectors $\mathbf{u}$ and the rotation vectors $\boldsymbol{\varphi}$ are given on the lateral face Σ; i.e.,

$$\mathbf{u}(x', x^3, t)\Big|_{\Sigma} = \mathbf{f}(x', x^3, t), \quad \boldsymbol{\varphi}(x', x^3, t)\Big|_{\Sigma} = \mathbf{g}(x', x^3, t).$$

Then the kinematic boundary conditions in moments of the Nth approximation with respect to the system of orthonormal shifted Chebyshev polynomials of the second kind can be represented as

$$\overset{(k)}{\mathbf{u}}(x',t) = \overset{(k)}{\mathbf{f}}(x',t), \quad \overset{(k)}{\boldsymbol{\varphi}}(x',t) = \overset{(k)}{\mathbf{g}}(x',t), \quad k=\overline{0,\,N}, \quad x' \in \partial\overset{(-)}{S}. \tag{6.17}$$

Here $\overset{(k)}{\mathbf{f}}(x',t)$ and $\overset{(k)}{\mathbf{g}}(x',t)$, $k=\overline{0,\,N}$, are known vector fields on $\partial\overset{(-)}{S}$ treated as the moments of known vector fields $\mathbf{f}(x',x^3,t)$ and $\mathbf{g}(x',x^3,t)$, respectively.

6.4. Physical boundary conditions in moments

Suppose that, on the lateral face Σ, the stress vectors $\mathbf{P}(x',x^3,t)$ and the couple stress vectors $\boldsymbol{\mu}(x',x^3,t)$ are given. Then, owing to the Cauchy formulas, the boundary conditions on the lateral face Σ can be written as

$$\mathbf{m}\cdot\underset{\sim}{\mathbf{P}}(x',x^3,t) = \mathbf{P}(x',x^3,t), \quad \mathbf{m}\cdot\underset{\sim}{\boldsymbol{\mu}}(x',x^3,t) = \boldsymbol{\mu}(x',x^3,t), \quad x' \in \partial\overset{(-)}{S}. \tag{6.18}$$

Here $\mathbf{m}$ is the unit normal vector at an arbitrary point on the lateral face.

Prior to obtaining the boundary conditions in moments, we derive several geometric relations on the lateral face for the new parametrization of the domain occupied by a thin solid, assuming that $\mathbf{h}\perp\overset{(-)}{S}$. Denoting by $d\Sigma$ an elementary area with one of the vertices at the point with coordinates (x^1,x^2,x^3) and with sides $d\mathbf{r}=\mathbf{r}_I\,dx^I$ and $\mathbf{h}\,dx^3=\mathbf{r}_3\,dx^3$, we obtain

$$\begin{aligned}
&d\boldsymbol{\Sigma} = d\Sigma\mathbf{m} = d\mathbf{r}\times\mathbf{h}\,dx^3 = \mathbf{r}_I\times\mathbf{r}_3\,dx^I\,dx^3 = \sqrt{g}\epsilon_{IJ}\mathbf{r}^I\,dx^J\,dx^3 = \sqrt{\overset{(-)}{g}}\,g_I^{K^-}\,\epsilon_{JK}\mathbf{r}^J\,dx^I\,dx^3,\\
&d\boldsymbol{\Sigma} = d\Sigma\mathbf{m} = \sqrt{g}\epsilon_{IJ}\mathbf{r}^I\,dx^J\,dx^3 = \sqrt{\overset{(-)}{g}}\,g_I^{K^-}\,\epsilon_{JK}\mathbf{r}^J\,dx^I\,dx^3,\\
&d\overset{(+)}{\boldsymbol{\Sigma}} = d\overset{(+)}{\Sigma}\overset{(+)}{\mathbf{m}} = \sqrt{\overset{(+)}{g}}\epsilon_{IJ}\mathbf{r}^{I^+}\,dx^J\,dx^3 = \sqrt{\overset{(-)}{g}}\,g_{I^+}^{K^-}\,\epsilon_{IJ}\mathbf{r}^{J^+}\,dx^I\,dx^3,\\
&d\overset{(-)}{\boldsymbol{\Sigma}} = d\overset{(-)}{\Sigma}\overset{(-)}{\mathbf{m}} = \sqrt{\overset{(-)}{g}}\epsilon_{IJ}\mathbf{r}^{I^-}\,dx^J\,dx^3.
\end{aligned} \tag{6.19}$$

The last two relations in (6.19) can be obtained in a similar way. Hence we can obtain them from the first relation for $x^3=1$ and $x^3=0$, respectively. Here $d\overset{(+)}{\Sigma}$ is a surface area element with one vertex at the point $\overset{(+)}{M}$ with coordinates $(x^1,x^2,1)$ and with sides $d\overset{(+)}{\mathbf{r}}=\mathbf{r}_{I^+}\,dx^I$ and $\mathbf{h}\,dx^3=\mathbf{r}_3\,dx^3$; $d\overset{(-)}{\Sigma}$ is a surface area element with one vertex at the point $\overset{(-)}{M}$ with coordinates $(x^1,x^2,1)$ and with sides $d\overset{(-)}{\mathbf{r}}=\mathbf{r}_{I^-}\,dx^I$ and $\mathbf{h}\,dx^3=\mathbf{r}_3\,dx^3$; $\overset{(+)}{\mathbf{m}}$ and $\overset{(-)}{\mathbf{m}}$ are the unit normal vectors at the points $\overset{(+)}{M}$ and $\overset{(-)}{M}$, respectively.

Next, from the first and third relations in (6.19) we obtain

$$d\Sigma = \overset{(-)}{\vartheta}\,\frac{\sqrt{g^{KL}\epsilon_{KI}\epsilon_{LJ}\,dx^I\,dx^J}}{\sqrt{g^{K^-L^-}\epsilon_{KI}\epsilon_{LJ}\,dx^I\,dx^J}}\,d\overset{(-)}{\Sigma} = \frac{\sqrt{g^{M^-N^-}\epsilon_{MK}\epsilon_{NL}g_I^{K^-}\,g_J^{L^-}\,dx^I\,dx^J}}{\sqrt{g^{K^-L^-}\epsilon_{KI}\epsilon_{LJ}\,dx^I\,dx^J}}\,d\overset{(-)}{\Sigma}. \tag{6.20}$$

Multiplying the first, second, and third relations in (6.19) by $\mathbf{r}_K$, $\mathbf{r}_{K^+}$, and $\mathbf{r}_{K^-}$, respectively, we obtain

$$d\Sigma m_I = \sqrt{g}\epsilon_{IJ}\,dx^J\,dx^3, \quad d\overset{(+)}{\Sigma}\overset{(+)}{m}_{I^+} = \sqrt{\overset{(+)}{g}}\epsilon_{IJ}\,dx^J\,dx^3, \quad d\overset{(-)}{\Sigma}\overset{(-)}{m}_{I^-} = \sqrt{\overset{(-)}{g}}\epsilon_{IJ}\,dx^J\,dx^3,$$

which implies that

$$d\Sigma m_I = \overset{(+)}{\vartheta}\,d\overset{(+)}{\Sigma}\overset{(+)}{m}_{I^+} = \overset{(-)}{\vartheta}\,d\overset{(-)}{\Sigma}\overset{(-)}{m}_{I^-}. \tag{6.21}$$

Now we find the boundary conditions in moments. First, note that (6.18) can be represented as

$$m_I\mathbf{P}^I = \mathbf{P}(x',x^3,t), \quad m_I\boldsymbol{\mu}^I = \boldsymbol{\mu}(x',x^3,t), \quad x' \in \partial\overset{(-)}{S}. \tag{6.22}$$

By multiplying each relation in (6.22) by $d\Sigma$ and by taking into account (6.21), we find

$$\overset{(-)}{m}_{I^-} g^I_{J^-} \mathbf{P}^{J^-} = \mathbf{P} \frac{d\Sigma}{\overset{(-)}{d\Sigma}} \overset{(-)}{\vartheta}{}^{-1}, \quad \overset{(-)}{m}_{I^-} g^I_{J^-} \boldsymbol{\mu}^{J^-} = \boldsymbol{\mu} \frac{d\Sigma}{\overset{(-)}{d\Sigma}} \overset{(-)}{\vartheta}{}^{-1}, \quad x' \in \partial \overset{(-)}{S}. \tag{6.23}$$

By introducing the notation

$$a(x', x^3) = \frac{d\Sigma}{\overset{(-)}{d\Sigma}} \overset{(-)}{\vartheta} = \frac{\sqrt{g^{M^- N^-} \epsilon_{MK} \epsilon_{NL} g^{K^-}_I g^{L^-}_J \, dx^I \, dx^J}}{\sqrt{g^{K^- L^-} \epsilon_{KI} \epsilon_{LJ} \, dx^I \, dx^J}} \overset{(-)}{\vartheta}{}^{-1}, \tag{6.24}$$

we can rewrite (6.23) in the form

$$\overset{(-)}{m}_{I^-} g^I_{J^-} \mathbf{P}^{J^-} = a(x', x^3) \mathbf{P}(x', x^3, t), \quad \overset{(-)}{m}_{I^-} g^I_{J^-} \boldsymbol{\mu}^{J^-} = a(x', x^3) \boldsymbol{\mu}(x', x^3, t), \quad x' \in \partial \overset{(-)}{S}. \tag{6.25}$$

By representing $a(x', x^3)$ as a series in x^3,

$$a(x', x^3) = \sum_{s=0}^{\infty} A_s(x')(x^3)^s, \quad A_s(x') = \frac{1}{s!} \left(\frac{\partial^s a}{\partial (x^3)^s} \right)_{x^3 = 0}, \tag{6.26}$$

and by taking into account the first relation in (3.15), from (6.25) we obtain the following boundary conditions of the rth approximation:

$$\overset{(-)}{m}_{I^-} \underset{(r)}{g}{}^I_{J^-} \mathbf{P}^{J^-} = a_{(r)}(x', x^3) \mathbf{P}, \quad \overset{(-)}{m}_{I^-} \underset{(r)}{g}{}^I_{J^-} \boldsymbol{\mu}^{J^-} = a_{(r)}(x', x^3) \boldsymbol{\mu}, \quad r \in \mathbb{N}_0, \quad x' \in \partial \overset{(-)}{S}, \tag{6.27}$$

$$a_{(r)}(x', x^3) = \sum_{s=0}^{r} A_s(x')(x^3)^s, \quad r \in \mathbb{N}_0.$$

By taking into account the first relations in (3.15) and (6.26) and by matching the coefficients of like powers of x^3 on the right- and left-hand sides, from (6.25) we obtain

$$\overset{(-)}{m}_{I^-} \underset{(s)}{A}{}^{I^-}_{J^+} \mathbf{P}^{J^-} = A_{(s)}(x') \mathbf{P}, \quad \overset{(-)}{m}_{I^-} \underset{(s)}{A}{}^{I^-}_{J^+} \boldsymbol{\mu}^{J^-} = A_{(s)}(x') \boldsymbol{\mu}, \quad s \in \mathbb{N}_0, \quad x' \in \partial \overset{(-)}{S}. \tag{6.28}$$

Relations (6.25) and (6.28) are equivalent, and relations (6.27) are equivalent to the first $r+1$ relations in (6.28). By applying the kth moment operator to (6.27), in view of (4.3), we obtain

$$\overset{(-)}{m}_{I^-} \overset{(k)}{\mathbf{M}} \big(\underset{(r)}{g}{}^I_{J^-} \mathbf{P}^{J^-} \big) = \overset{(k)}{\mathbf{M}} (a_{(r)} \mathbf{P}), \quad \overset{(-)}{m}_{I^-} \overset{(k)}{\mathbf{M}} \big(\underset{(r)}{g}{}^I_{J^-} \boldsymbol{\mu}^{J^-} \big) = \overset{(k)}{\mathbf{M}} (a_{(r)} \boldsymbol{\mu}), \quad r = \overline{0, N}, \quad x' \in \partial \overset{(-)}{S}. \tag{6.29}$$

By taking into account (6.28), from (6.29) we obtain the relations

$$\overset{(-)}{m}_{I^-} \underset{(s)}{A}{}^{I^-}_{J^+} \overset{(k)}{\mathbf{P}}{}^{J^-} = A_{(s)} \overset{(k)}{\mathbf{P}}, \quad \overset{(-)}{m}_{I^-} \underset{(s)}{A}{}^{I^-}_{J^+} \overset{(k)}{\boldsymbol{\mu}}{}^{J^-} = A_{(s)} \overset{(k)}{\boldsymbol{\mu}}, \quad s = \overline{0, r}, \quad k = \overline{0, N}, \quad x' \in \partial \overset{(-)}{S}, \tag{6.30}$$

which can also be obtained by applying the kth moment operator to relations (6.28).

We note that, on the basis of (6.28), we can exclude the moments of desired and known variables whose order exceeds N from (6.29). Then we obtain relations which we call the static boundary conditions in moments of the (r, N)th approximation. They are equivalent to (6.30), and hence it is expedient to consider relations (6.30) as the static boundary conditions in moments of the (r, N)th approximation.

6.5. Boundary conditions of heat content in moments

We consider the boundary conditions of the first (Dirichlet), second (Neumann), and third (heat exchange with environment according to the Newton law) kinds [25] and derive the corresponding boundary conditions in moments from them.

6.5.1. Boundary conditions of the first kind in moments. In this case, temperature is given on part $\Sigma_q \subseteq \Sigma$ of the lateral face Σ:

$$T(x', x^3, t)\Big|_{\Sigma_q} = T_0(x', x^3, t).$$

Hence, by analogy with (6.17), the desired boundary conditions of the first kind in moments have the form

$$\overset{(k)}{T}(x', t) = \overset{(k)}{T}_0(x', t), \quad k = \overline{0,\, N}, \quad x' \in \partial \overset{(-)}{S}_q \subseteq \partial \overset{(-)}{S}, \tag{6.31}$$

where $\overset{(-)}{T}_0(x', t)$, $k = \overline{0,\, N}$, are known moments of the known scalar field $T_0(x', x^3, t)$.

6.5.2. Boundary conditions of the second kind in moments. In this case, the following condition is satisfied on part $\Sigma_q \subseteq \Sigma$ of the lateral face:

$$\mathbf{m} \cdot \mathbf{q}(x', x^3, t)\Big|_{\Sigma_q} = q_0(x', x^3, t) \quad (m_I q^I\Big|_{\Sigma_q} = q_0).$$

Hence, omitting the derivation, by analogy with (6.30) we obtain the desired conditions in the form

$$\overset{(-)}{m}_{I^-} \underset{(s)}{A}{}^{I^-}_{J^+} q^{J^-}(x', t) = A_{(s)} \overset{(k)}{q}_0(x', t), \quad s = \overline{0,\, r}, \quad k = \overline{0,\, N}, \quad x' \in \partial \overset{(-)}{S}_q \subseteq \partial \overset{(-)}{S}. \tag{6.32}$$

Relations (6.32) will be called the boundary conditions of heat content of the second kind in moments of the (r, N)th approximation.

6.5.3. Boundary conditions of the third kind in moments. In this case, the boundary conditions can be represented as

$$\mathbf{m} \cdot \mathbf{q}(x', x^3, t)\big|_{\Sigma_q} = \beta(T_c - T\big|_{\Sigma_q}) \quad (m_I q^I\big|_{\Sigma_q} = \beta(T_c - T\big|_{\Sigma_q})). \tag{6.33}$$

Then, by analogy with (6.32), from (6.33) we obtain the following expressions for the desired conditions:

$$\overset{(-)}{m}_{I^-} \underset{(s)}{A}{}^{I^-}_{J^+} q^{J^-}(x', t) = A_{(s)} \beta(\overset{(k)}{T}_c - \overset{(k)}{T}), \quad s = \overline{0,\, r} \quad k = \overline{0,\, N} \quad x' \in \partial \overset{(-)}{S}_q \subseteq \partial \overset{(-)}{S}. \tag{6.34}$$

Relations (6.34) will be called the boundary conditions of heat content of the third kind in moments of the (r, N)th approximation.

When writing (6.34), we have assumed that the convective heat transfer coefficient β is independent of x^3. If β depends on x^3, then, to find the kth moment of the right-hand side in (6.33), it is necessary to use the first relation in (4.10) for $s = 0$. We also note that it is possible to consider boundary conditions of the form more general than the above [25]. If we need to obtain systems of equations of motion and heat influx in moments from some other representations similar to (3.18) and (3.21), then we should represent the systems of boundary conditions of physical and heat content (conditions of the second and third kinds) in moments in an appropriate form. For brevity, we do not consider these problems in the present paper.

6.6. Initial conditions in moments

If nonstationary problems are studied, then it is necessary to pose some initial conditions at some time instant $t = t_0$. Suppose that the initial conditions for the nonstationary (dynamic) problem of the moment MTDS are given in the form

$$\begin{aligned} &\mathbf{u}(x', x^3, t)\big|_{t=t_0} = \mathbf{u}_0(x', x^3), \qquad \partial_t \mathbf{u}(x', x^3, t)\big|_{t=t_0} = \mathbf{v}_0(x', x^3), \\ &\boldsymbol{\varphi}(x', x^3, t)\big|_{t=t_0} = \boldsymbol{\varphi}_0(x', x^3), \qquad \partial_t \boldsymbol{\varphi}(x', x^3, t)\big|_{t=t_0} = \boldsymbol{\omega}_0(x', x^3), \end{aligned} \tag{6.35}$$

and, for the nonstationary thermal conduction problem, the initial condition is given in the form

$$T(x', x^3, t)\big|_{t=t_0} = T^0(x', x^3). \tag{6.36}$$

From (6.35), for the desired initial conditions in moments we obtain the expressions

$$\begin{aligned} &\overset{(k)}{\mathbf{u}}(x',t)\big|_{t=t_0} = \overset{(k)}{\mathbf{u}}_0(x'), \qquad \partial_t\overset{(k)}{\mathbf{u}}(x',t)\big|_{t=t_0} = \overset{(k)}{\mathbf{v}}_0(x'), \\ &\overset{(k)}{\boldsymbol{\varphi}}(x',t)\big|_{t=t_0} = \overset{(k)}{\boldsymbol{\varphi}}_0(x'), \qquad \partial_t\overset{(k)}{\boldsymbol{\varphi}}(x',t)\big|_{t=t_0} = \overset{(k)}{\boldsymbol{\omega}}_0(x'), \quad k=\overline{0,\,N}, \quad x'\in\partial\overset{(-)}{S}. \end{aligned} \tag{6.37}$$

In a similar way, from (6.36) for the nonstationary thermal conduction problem we obtain the following system of initial conditions in moments:

$$\overset{(k)}{T}(x',t)\big|_{t=t_0} = \overset{(k)}{T}{}^0(x'), \quad k=\overline{0,\,N}, \quad x'\in\partial\overset{(-)}{S}. \tag{6.38}$$

We note that (6.37) and (6.38) form the system of initial conditions in moments of the Nth approximation for the dynamic problem of moment thermomechanics of thin deformable solids (TMTDS).

7. CLASSIFICATION AND STATEMENT OF PROBLEMS IN THE THEORY OF THIN SOLIDS

The classification and statement of problems in both the moment and the classical theory of thin solids is performed in the same way as in mechanics of deformable solids (MDS) [25].

In contrast to MDS, in the case considered here, the approximate CE and the equations of motion and thermal conduction in moments are considered. In this case, the boundary conditions are posed on the boundary contour of the main base surface in moments.

7.1. Classification of MTDS problems in moments

For boundary conditions and the corresponding boundary value problems in the moment theory, the following classification is accepted, which we formulate in the form of definitions.

Definition 7.1. If on the boundary contour $\partial\overset{(-)}{S}$ only the kinematic boundary conditions in moments of the Nth approximation (6.19) are given, then such conditions are called the boundary conditions of first kind and the MTDS problem using these conditions is called the first boundary value problem.

Definition 7.2. If on the boundary contour $\partial\overset{(-)}{S}$ only the static boundary conditions in moments of the (r,N)th approximation (6.32) are given, then such conditions are called the boundary conditions of the second kind and the MTDS problem using these conditions is called the second boundary value problem.

Definition 7.3. If, on one part $\partial\overset{(-)}{S}_1$ of the boundary contour $\partial\overset{(-)}{S}$, the kinematic boundary conditions in moments of the Nth approximation (6.19) are given and on the other part $\partial\overset{(-)}{S}_2$ of this contour, the static boundary conditions in moments of the (r,N)th approximation (6.32) are given, $\partial\overset{(-)}{S}_1\cup\partial\overset{(-)}{S}_2=\partial\overset{(-)}{S}$, $\partial\overset{(-)}{S}_1\cap\partial\overset{(-)}{S}_2=\varnothing$, then such conditions are called mixed boundary conditions and the MTDS problem using these conditions is called the mixed boundary value problem of MTDS.

It should be noted that, in the case of dynamical problems, at a certain time instant $t=t_0$, the initial conditions in moments of the Nth approximation (6.37) must be given. If the thin solid is unbounded, then some conditions in moments must be posed at infinity. Excluding the characteristics of the moment theory from the above definitions, we obtain the corresponding definitions for the classical MTDS.

7.2. Statement of TMTDS problems in moments

We consider the statements of coupled and uncoupled dynamical problems in moments of the (r,N)th approximation of the moment TMTDS, as well as of the unsteady temperature problem in moments the (r,N)th approximation of the moment TMTDS, and discuss how to obtain some other special cases of problem statements from them.

The statement of the coupled dynamical problem in moments of the (r,N)th approximation of the moment TMTDS includes:

1) the system of equations of motion in moments of the (r, N)th approximation of the moment TMTDS;

2) the system of equations of the heat influx in moments of the (r, N)th approximation of the moment TMTDS;

3) the system of CE in normalized moments of the stress and couple stress tensors of the (r, N)th approximation of the moment TMTDS or the system of CE in moments of the (r, N)th approximation of the moment TMTDS for the simplified reduction scheme;

4) the system of Fourier's thermal conduction laws in normalized moments of the heat influx vector of the (r, N)th approximation of the moment TMTDS or the system of Fourier's thermal conduction laws in moments of the (r, N)th approximation of the moment TMTDS for the simplified reduction scheme;

5) depending on the type of boundary value problems, one of the following systems of boundary conditions in moments:

5a) the system of kinetic boundary conditions in moments of the Nth approximation (6.17) for the first boundary value problem and a set of systems of boundary conditions of heat content of three types in moments (6.31), (6.32), or (6.34);

5b) the system of static boundary conditions in moments of the (r, N)th approximation of the moment TMTDS (6.30) for the second boundary value problem and a set of systems of boundary conditions of heat content of three types in moments (6.31), (6.32), or (6.34);

5c) the system of kinematic boundary conditions in moments of the Nth approximation (6.17) on a part of the boundary contour and the system of static boundary conditions in moments of the (r, N)th approximation of the moment TMTDS (6.30) on the other part of the boundary contour for the mixed boundary value problem and a set of systems of boundary conditions of heat content of three types in moments (6.31), (6.32), or (6.34);

6) the systems of initial conditions for the kinematic (6.37) and heat (6.38) intensities in moments of the Nth approximation.

If the system of heat influx equations in moments of the (r, N)th approximation does not contain mechanical characteristics (the moments of the stress tensors $\overset{(k)}{\underset{\sim}{\mathbf{P}}}$ and of the couple stress tensors $\overset{(k)}{\underset{\sim}{\boldsymbol{\mu}}}$), then the nonstationary temperature problem in moments of the (r, N)th approximation is studied separately. This problems includes:

1) the system of heat influx equations in moments of the (r, N)th approximation without mechanical characteristics;

2) the system of Fourier thermal conduction laws in normalized moments of the heat influx vector of the (r, N)th approximation of the moment TMTDS or the system of Fourier thermal conduction laws in moments of the (r, N)th approximation of the moment TMTDS for the simplified reduction scheme;

3) a set of of systems of boundary conditions of heat content of three types in moments (6.31), (6.32), or (6.34);

4) the system of initial conditions of heat content in moments of the Nth approximation.

In this case, the dynamic problem in moments of the (r, N)th approximation of the moment TMTDS splits into the following two problems: the nonstationary temperature problem in moments of the (r, N)th approximation whose solution determines the temperature field which, in what follows, is assumed to be known and the dynamic problem in moments of the (r, N)th approximation of the moment TMTDS in nonisothermal processes with known temperature field, which includes:

1) the system of equations of motion in moments of the (r, N)th approximation of the moment TMTDS in nonisothermal processes with known temperature field;

2) the system of CE in normalized moments of the stress and couple stress tensors of the (r, N)th approximation of the moment TMTDS with known temperature field or the system of CE in moments of the (r, N)th approximation of the moment TMTDS for the simplified reduction scheme;

3) depending on the type of the boundary value problems, one of the following systems of boundary conditions in moments:

3a) the system of kinematic boundary conditions in moments of the Nth approximation (6.17) for the first boundary value problem;

3b) the system of static boundary conditions in moments of the (r, N)th approximation of the moment TMTDS for the second boundary value problem;

3c) the system of kinematic boundary conditions in moments of the Nth approximation (6.17) on a part of the boundary contour and the system of static boundary conditions in moments of the (r, N)th approximation of the moment TMTDS (6.30) on the other part of the boundary contour for the mixed boundary value problem.

4) the system of kinematic initial conditions in moments of the Nth approximation (6.37).

Problems for nonisothermal processes, which are divided into the temperature problem and the TMTDS problem with known temperature field, are called uncoupled TMTDS problems [25].

Thus, in the preceding we have given the statements of coupled and uncoupled dynamic problems in moments of the (r, N)th approximation of the moment TMTDS as well as of the nonstationary temperature problem in moments of the (r, N)th approximation. From these statements of the problems, one can readily obtain statements of the corresponding static and quasistatic problems and also, by considering various values of r and N, statements of the problems in moments of the desired approximations. Furthermore, one can obtain statements of the problems for isothermal processes. Finally, if in all statements of the problems given or mentioned above one neglects the moments of couple stresses and of the internal rotation vector, then one obtains the corresponding statements of the problems in moments of the (r, N)th approximation in the classical TMTDS and MTDS. Statements of the problem of the moment theory for an arbitrary anisotropic material under the classical parametrization of the domain occupied by the thin solid on the basis of Legendre polynomials was considered in [27].

The present paper essentially extends Vekua's theory [23] to the moment TMTDS for an arbitrary anisotropic material; in contrast to [23], we use Chebyshev polynomials of the second kind instead of Legendre polynomials, which permits one to obtain more concise and general relations than in the case where Legendre polynomials are used.

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